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Looking at Solid Geometry Through Perspective

By Ethel Spearman, *Sanger, California*

"Much attention should be given to the visualization of spatial figures and relations, to the representation of three-dimensional figures on paper, and to the solution of problems in mensuration," says the Joint Commission among its suggestions on solid geometry in its report on "The Place of Mathematics in Secondary Education."* These ideas are incorporated in the notion of perspective. Perspective may be regarded as a practical means for securing a rigorous reciprocal metrical relationship between the shapes of objects as definitely located in space and their pictorial representation. It may be regarded as the rationalization of sight. This Commission further points out that "the problems in mensuration offer opportunity for correlation of solid geometry with arithmetic, algebra, and trigonometry. Part of the importance of the geometry of the sphere comes from the perspective that it makes possible."

The purpose of this paper is to organize some ideas, partly by drawings and partly by discussion, in keeping with the above recommendation about the rationalization of sight. The whole theory of perspective can be developed from a single basic example, as a railroad train moving over a straight track. As the train moves farther and farther away its dimensions apparently become smaller and smaller. Its speed seems to decrease, for the space over which it travels in a given time seems to be less and less as the train gradually recedes. Again the railroad track is of uniform width, yet in appearance the rails seem to meet in the distance. Lines and forms seem to change in size and shape as they occupy different positions in the picture. The distance and position of objects affect both their distinctness and apparent form.

Linear perspective in a picture is obtained by drawing upon the perspective plane three lines; the base or ground line, the horizon line, and the vertical line. The base line is considered as the base line of the imaginary vertical plane called the picture plane. The horizon line represents the ordinary position of the sensible horizon. It is, in nearly all cases, supposed to be level with the spectator's eye. The vertical line which is drawn from the supposed position of the sketcher perpendicular to the ground and horizon line, meets the latter in a point which is called the point of sight or center of the picture. The vertical line is merely a mechanical aid to the construction of the picture. All

* The Joint Commission is composed of fourteen members, seven from the Mathematical Association of America and seven from the National Council of Teachers of Mathematics. This report, "The Place of Mathematics in Secondary Education," Final Report of this Commission, was published as the 15th Yearbook of the National Council of Teachers of Mathematics in 1940.

vertical lines in nature are parallel to it in the picture.

The points of distance are two points in the horizon line on each side of the point of sight. In all cases, the two points of distance are about twice as far apart as the eye is from the picture. In the case of the projection of a square, the distance from the point of sight to one distant point is arbitrarily taken equal to one and one half times the diagonal of the square. One important use of the points of distance is to define the distance of objects in a row from each other.

The simplest form of the perspective problem is how to throw a square into a geometrical projection. Figure 1 is included here to show the lines and points

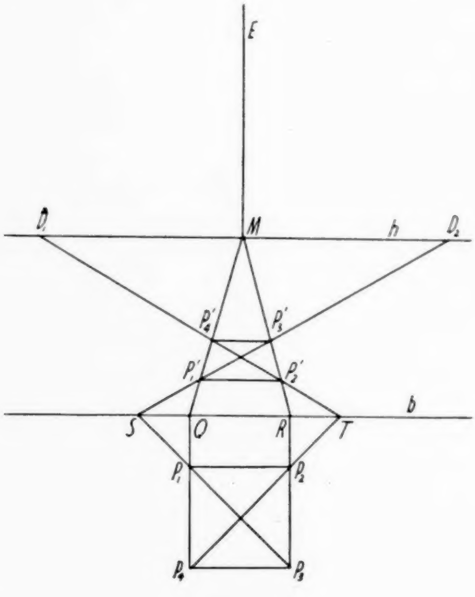

FIG. 1

already discussed, and to show the square, $P_1'P_2'P_3'P_4'$, in the vertical plane, the perspective of the square, $P_1P_2P_3P_4$, in the ground plane. In the figure, *h* is the horizon line; *b*, the base line; and *EM*, the vertical line. The four points are: *E*, the observer; *M*, the point of sight, D_1, and D_2, the distance points.

An example is included to illustrate parallel projection. In Figure 2, the image of the triangle in the object plane, 0, is depicted in the picture plane, *R*.

The relation of similarity between object and image can thus be shown by perspective.

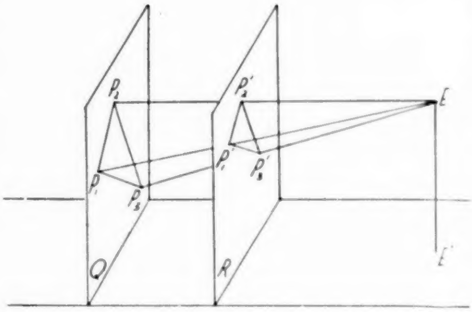

FIG. 2

In solid geometry a plane passed through the pyramid parallel to the base plane forms a similar figure. It can be pointed out that the vertex of the pyramid might be considered the observer, and a lesson in perspective can be observed.

A cube is known to have twelve equal edges and six perfect squares as faces. When viewed, the edges do not appear to be equal, nor do the faces appear to be squares. Figure 3 is an isometric drawing

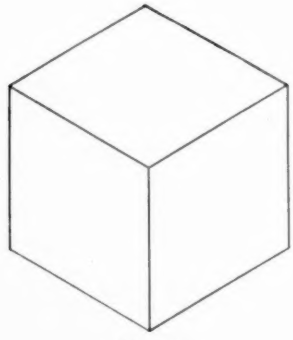

FIG. 3

used to show dimensions in their true size to the scale of the drawing. It is useful to indicate the size of the object. This is purely an arbitrary method of representation which gives a distorted picture of the object. The isometric drawing should not be confused with the true perspective.

On the contrary, one can construct a cube in perspective. A cube is constructed in the ground plane and projected in the vertical plane as shown in Figure 4. For

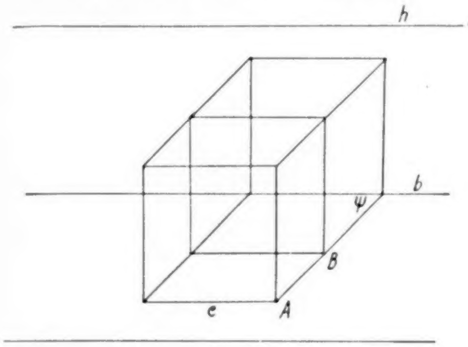

FIG. 4

this construction: e is the edge of the cube; b, the base line; h, the horizon line; ψ equals 45°; and AB equals $\frac{1}{2}e$. The lines parallel to AB in the cube are projected on the vertical plane. These lines project into four points. The four points are joined to make the square (in the vertical plane) which is the projection of the cube on the vertical plane.

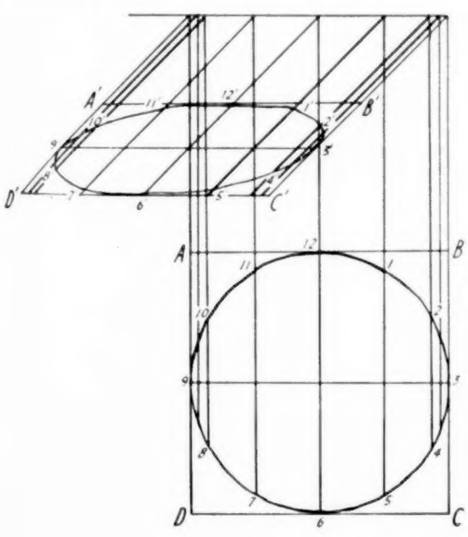

FIG. 5

The adjacent figure depicts the perspective representation of a circle viewed at an angle of 45° in the perspective plane. Since a circle may be considered a polygon of an infinite number of points, and since the perspective of any point in the curve may be found, it follows that every point can be found. The more points that can be designated without confusion, the more correct will be the representation. The perspective of the circle is an ellipse.

Figure 6 is the perspective representation of a circle viewed directly in front of and touching the perspective plane. Here the distance points, D_1 and D_2, from the point of sight, M, are taken equal to the diagonal of the square circumscribed about the circle to be projected.

By using the pattern of an ellipse such as was constructed in Figure 5, one may depict a sphere as it is viewed in the per-

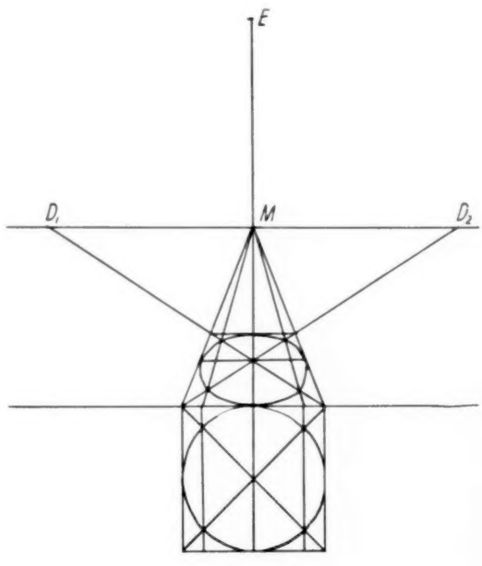

FIG. 6

spective plane. By means of the pattern of an ellipse, two ellipses were traced off with their axes perpendicular to each other. $OW = OC = OP$. N and S are focal points. AB is the major axis. CD is the minor axis. The ellipse with axes AB and CD is constructed. Thus the sphere is constructed.

The emergency of the ideas that led to the rationalization of sight is due to the mechanics of the perspective schemes of Alberti, Pelerin (known as the Viator),

and Dürer whose efforts towards that rationalization received its first expression in

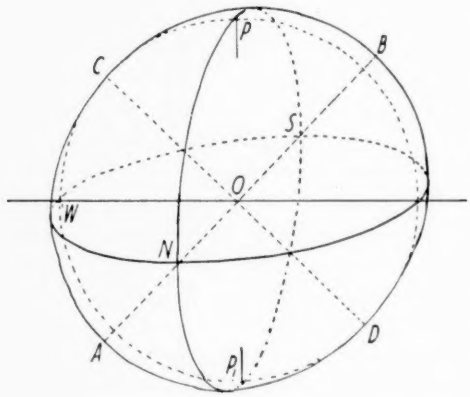

Fig. 7

Italy, France and Germany. Leone Battista Alberti was the first to discover a simple but logical scheme for pictorial perspective. His scheme marks the effectual beginning of the substitution of visual for tactile space awareness. His book is generally acknowledged to be the earliest statement of a logically coherent and pictorially adequate scheme of perspective representation. By common agreement the three outstanding Renaissance texts on perspective are: The Della Pittura Libri Tre by Alberti, 1435–36; De Artificiali Perspective by Viator, 1505; and Unterwegsung der Messung by Dürer, 1529. It is to be noted that all great painters understood the principles of perspective.

The foregoing remarks are not intended to be a comprehensive study on perspective, but they serve to point out the fact that solid geometry gives an opportunity for emphasizing perspective.

A Reorganization of Geometry for Carryover

By Harold D. Aten
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"I ENROLLED in this course merely to complete the college entrance requirements . . . Now I wish that I could study geometry all the rest of the time I am in high school." The fifteen-year-old writer of the preceding statement had little interest or ability in mathematics. Early in the course he tried to explain a postulate by a highly-prized "picture of one." With I.Q. (Terman) 98, he ranked in the third quartile of eighty-five tenth grade pupils who formed our experimental group. He kept a detailed notebook of theorems and daily assignments, written up in his own words. At the end of the year he confided that he had never seen inside a geometry book. He took the Cooperative Plane Geometry test, Revised Series Form Q, of the American Council of Education with a score of 25.5, about 40 per cent above the standard for the country as a whole.

Readaptations of plane geometry tried out with our classes contemplated no change in so far as end results were concerned. Rather the learning situation was redirected to secure better pupil insight into an appreciation of how geometry contributes to his life purposes. Recognizing that the conventional course had failed to elicit the fullest interest and response so vital to learning, and further that this the most perfect of the sciences did have a high functional potential for all youngsters, means were sought to point the teaching toward those thinking patterns so useful to all who are privileged to live in a democratic state. If geometry is essentially a method of thought and its chief contribution is to be the implementation of thinking, situations need be created in which the transfer may occur. For the process is not automatic; the teacher must set the stage, suggest and defend the goals, observe the results, appraise the new abilities.

Before discussing the specific readaptations made in the presentation of subject matter, let us consider some of the assumptions upon which the reordering seems to rest. For well we teachers of the subject know that no conclusion can transcend in validity its supporting postulates.

There is a well-known causal liaison between interest and effort in learning. An implied corollary is the assertion that accomplishment of worth is possible only in those tasks which the pupil is happy to undertake. Very often pupils are hostile; they approach geometry with prejudicial fear; struggle through the maze; sigh with relief when the hurdle is cleared; recall with delight (almost fiendish glee) how they beat the game by memorizing the theorems. More direct adaptation to environment is often had by the device of copying the neighbor's home work! The ever-widening panorama of subject matter in which the pupil may register a choice, as well as the increased latitude he is coming to exercise in the making of his choice, concern the future role of geometry as a school experience.

Little thought has been given to geometry as a terminal subject. Irrespective of whether or not it should be, statistics show a tendency for it so to become in a majority of cases. In California in 1940 the number of engineering degrees awarded by colleges was less than one per cent of the enrollment in high school plane geometry, at the same time. Perhaps the difference in needs is not a function of this expectancy. Nevertheless, it is surely worthy of consideration.

Again, Euclid's content, and his well-knit sequence have enjoyed a rather unique and sedate course through the educational currents and eddies of some twenty-six centuries. During the first twenty-five of these centuries no voice had

been raised to question the assumptions of the physical universe which his imagination had contrived. We know now that the space-time continuum is intuitively sensed even by a preschool child, long before he is confronted by a set of eternal verities which deny the fourth dimension. If we reply that it is not orientation in the physical world that we seek, but rather association with an ordered array of logical deduction, we perforce confront the other horn of a dilemma: the "ordered array" is a *finished product.* This abstracts its most vital element in the learning process.

Modern educational theory increasingly stresses the goal of clear thinking. Mathematicians have no patent on the process, but they do have a direct and important contribution to make. We know that the pupil does not think scientifically, mathematically, socially: he thinks well or ill, and all his experiences induce their fillip to that end. In the unique rigor of geometric proof the scientific method attains its purest type. Careful definition, exact expression, precise conclusions, relevancy, proof beyond a reasonable doubt—these are the peculiar outcomes of the teaching of geometry; likewise they form "the horn and the hoof, the haunch and the hump" of critical thought. Deductive thinking absorbs two propositions in the reasoning machine and grinds out a new truth. Induction aids the process, by appraising the like and unlike elements in different things. Call the result correct thinking, logical thinking, the scientific method, postulational thinking, autonomous thought, or what-not: they all refer to the same ability: an ability that cuts across all subject fields, containing elements that are automatic in all thinking, yet possessing a common core which may be made to transfer into those situations in which the learner can identify the like elements involved.

It is no more strange that Euclid knew nothing of psychology than it is that he conceived the world as being flat. The very perfection of his great work has rendered it uncommonly resistant to those changes which contemporary subjects have continuously undergone. If it has been immune to the annoying eddies, perhaps it has as truly withstood the emending current. That there is important work to be done, and that the surface is now but barely scratched, cannot escape the teacher who aspires to have his pupils see the inspiration and lasting satisfactions which he feels belong peculiarly to the study of geometry.

Challenged to help us tie in the experiences of geometry with what they hoped to become and what society expected them to become, members of the experimental classes responded with ardent and gratifying zeal. The teacher-pupil planned reorganization was carried along informally, with changes as their need was felt. The classes agreed it was their privilege to learn to think clearly, and a new interest was awakened. They contemplated the bridge we hoped they would help to build, and undertook to make the highway from geometry to life a real and a useful one. In the process two considerations have been constantly to the fore: the pupil was to be permitted to *discover* all he could under discreet guidance, and the geometry learned was to be immediately used in *nongeometric* situations.

After some time in which the nongeometric material was more or less promiscuously piecemealed into the geometric content, there began to appear a guiding principle for reorganization. We supplanted the orthodox "What is the form?" and "How does it grow from the basic assumptions?" with "How does it contribute to straight thinking?" Units were built around the thinking process. The pattern of logical thought displaced the pattern of spatial concepts. In this process, none of the theorems were lost; the sequence was readapted on a better psychological basis. The rearrangement was complete enough that the few who secured books for themselves soon had difficulty in finding the place and discarded them.

This had been foreseen. We had called the course "functional geometry," and the pupils were puzzled. They abbreviated this to "fun geometry," and we were puzzled. Now we fully understand each other, and the course is simply "plane geometry" until we can be sure that the term we use will carry a like implication to all.

Readaptations of procedure have been made without abruptness and under conditions of pupil cooperation in planning. There was always a framework of established procedure into which retreat could be made as required. Two years' try-out leaves eminent satisfaction with most of the changes made. The following changes seem sufficiently confirmed in practice:

1. The displacement of superposition proofs.
2. The discovery of assumptions and theorems by pupils from assigned given data.
3. The discovery of the proof by the students, with a few appropriate models, and flexible suggestions.
4. The assignment of originals without a "to-prove," the pupil to find and prove what he can from the given data.
5. The evolution of figures with proofs.
6. The organization of units about aspects of thinking. The theorems studied in each unit then become assimilative material directed at the mastery of a specific phase of straight thinking.
7. The incorporation into this assimilative material of parallel life situations using the same principles.
8. The evaluation of the learning in nongeometric as well as in geometric situations.
9. The introduction and use of inductive proof.
10. The use of notebooks in lieu of a text.

The testing program was first directed to insure that none of what is commonly understood as geometric learning was being lost in the process. Attention was then directed to evaluating the carryover into life situations. The difficulty as well as the pertinence will be readily recognized. Working with the staff in the Eight Year Study, giving their tests, analyzing results, and constructing new tests to meet newly defined abilities, leaves the feeling that the task, while difficult enough, is not insuperable. It is everyday practice for the teacher to appraise the pupil's ability to think. This appraisal grows from some observation of what the pupil does, and is perforce largely subjective. Measuring the thinking is not a new thing for the teacher; it is merely an attempt to render the opinion which is automatically registered more objective, and hence to place it in line for appropriate action. The crude instrument of measurement becomes increasingly precise, as sources of error are uncovered and removed.

In line with these considerations there were listed first sixty or seventy mental abilities ordinarily thought of as characterizing one who thinks clearly. These were allocated to the eleven units of the course, according to the specific objectives. Parallel tests were then devised to record the accomplishment of pupils in the mastery of each unit, both as to the use of the geometric principles included, and as to the facility with which the new proficiency carries over into analogous life situations. The general scope of the latter type of tests will appear from the following list of situations which were set up for scoring and interpretation. The pupil is directed to:

1. Find hidden assumptions in an argument.
2. Classify types of thinking.
3. Distinguish given data and conclusion.
4. Check key words needing definition.
5. Determine the relevance of evidence respecting a conclusion.
6. Judge the direction of evidence with respect to a conclusion.
7. Find implications of given data.
8. Determine the fulfillment of the law

of identity, and the law of causality.
9. Identify omitted relevant factors in argument.
10. Match general principle with specific instance.
11. Connect data in the if-then form.
12. Associate appropriate step and reason.
13. Find the starting point of proof.
14. Recognize all possibilities in indirect proof.
15. Recognize appropriate elimination by contradiction, and by exhaustion in indirect proof.
16. Recognize inadequate sampling.
17. Recognize false analogy.
18. Determine the propriety of generalization from observed data.
19. Determine fulfillment of necessary and sufficient conditions in proof.
20. Identify the more common fallacies in argument.

A noteworthy result of the reorganized course has been the spontaneous enthusiasm and response from the groups concerned. There is evidence also of better accomplishment, especially from the pupils whom we generally find rather slow in the work. The method is without doubt doing something for them. It may well be that the flexibility in assignments, as well as the incorporation of at least some material which does not appear too far afield to them, permits these pupils to find a certain sense of accomplishment, as a springboard to greater things. There is an unusually fine spirit of independent study and individual responsibility in the classes. How much of this is due to the techniques employed and how much to the consciousness that they are being directed in an experimental study it is impossible to say. It is at least some reassurance to find groups studying geometry at this age, and enjoying it, trying to improve their statements of discovered relationships, eager to explain or hear an original proof. It has been said that it is only in the armies of a democracy that the soldiers, having occupied the trench they were assigned to capture, would then have a look for themselves to see if anyone was in the trench beyond.

It is hoped to continue and to extend the practices here set forth. It is truly a long road, but were the end in plain sight, perhaps the fun would depart from the journey. We have directed our efforts in the hope that there may be more who come into the classrooms with the feeling, not of having borne upon their senses the enjoinder to attend well to venerable dicta and undying truths of the ages, but rather expecting an invitation to share in a venture that is planned so that theirs may be fuller lives. It is for us to continue to work over the products of the present levels, lifts and stoops, but not unmindful that the horizon is beyond, and truly there is still "gold in them thar hills!"

We cannot fully know now, and perhaps it will never be given us to know, how vitally the truths we impart will enter into the lives of these pupils. We do know, however, and it must always bear strongly upon our teaching, that mathematical thinking will play a very real part in the decisions of their individual lives, and in the evolution of the social organism of which they form a part. In this course, which may well be for many the last formal contact with a systematic study of mathematics, it is to be hoped that we have left a little deeper faith in how it may help them make their decisions more wisely, and impel them to grasp, and to hold fast, that which can be demonstrated as true beyond the peradventure of reasonable doubt. And, although it may not show objectively in the pay check, it is something to know that the customers are satisfied, and to be able to find among the various knocks, one who will say, "Now I wish that I could study geometry all the rest of the time that I am in high school."

The Challenge of the Bright Pupil

By Lillian Moore, *Far Rockaway High School, New York, New York*

There is so much time, energy and effort being expended to develop a mathematics course for the slow pupil, to select from arithmetic, algebra, geometry, trigonometry those units which will most benefit the non-academically minded pupil, entering the high schools in increasing numbers, that we are apt to forget that the bright pupil needs as much individual attention, adjustment, revised methods and content as the slow pupil. We are, also, likely to overlook the fact of supreme importance that it is more vital to provide for the maximum development of the powers and abilities of the bright pupil than of the mentally slower one.

As teachers in a public, tax-supported school system our foremost duty is to support, preserve and further our representative democracy, especially at the present time when democracy is being assailed from within and without our borders by the advocates of fascism, nazism or communism. In advancing this support, mathematics teachers have an unparalleled opportunity to select and train the future leaders in our democracy.

The present neglect and disregard for bright pupils is unfortunate, to say the least. In a big city, such as New York, they are being taken care of in a separate school. The Bronx High School of Science makes provision for boys with special aptitudes along scientific lines. The High School of Music and Art specializes in pupils with high artistic or musical capabilities. In a large academic high school pupils may be segregated according to their intelligence into slow, normal and bright groups, with corresponding changes and adjustments made in presentation methods, modified curriculum, rate of progress, and teacher-pupil relationships.

Such organization plans are possible and are being followed in large schools and in large school systems. But what of the bright pupil in the public high school of small or average size? The teacher in such a school can read oceans of literature on the challenge of the slow pupil, innumerable magazine articles on simplifying mathematics content, but very few on aid to bright pupils. There is certainly no objection to revising syllabi to take care of pupils who are preparing for business or the factory instead of for college, but the pendulum has swung too far in this direction. The content has been simplified to such an extent that there remains little to attract the interest and effort of the pupil with a high intelligence quotient.

My nephew is considerably annoyed because his geometry teacher is not covering enough ground. The class of which he is a member spent two weeks on constructions which he performed during the weekend. Why should one pupil sit in class wasting time? Teachers should expend some effort in providing maximum assignments for brighter pupils, requesting them to assist the slower members of the class, and in assigning special reports on relevant history of mathematics or mathematicians. I recall an exceedingly interesting report of one Jewish pupil on Gematria. One city high school is programming a non-credit class for exceptional mathematics students in a teacher's free period. In this class analytic geometry, differentiation, integration, determinants, number theory, theory of equations are considered. The pupil is given work which taxes his ability. The content of the course is not restricted by syllabus requirements nor preparation for final State examinations. We should not allow the influx into the secondary schools of pupils with low intelligence quotients to result in a leveling-down process in mathematics teaching. Leaders are not developed by allowing them to drift along without effort. They are developed by giving them work worthy of their ability,

by affording them the thrill of achievement which results from solving a difficult problem unaided.

It is fashionable these days to decry scholarship, to belittle scholastic accomplishment, to stress the social aspects of extra-curricular work, to substitute assemblies for class work, to train exclusively for associational living, to follow an activity program. One must be progressive! The schools should prepare for social and community life, but such objectives are valueless if the individual's own capacities are not developed to the highest possible degree. A recent newspaper report described the lowering of school standards in Nazi Germany due to the interruption of school work by patriotic assemblies and the social demands of political youth organizations. American schools should educate for democracy, but the safest course to follow in so doing is to develop leaders. These pupils should be leaders of thought, students who can reason, who can reach a conclusion as the result of weighing all sides of a question, who have developed the scientific attitude, who are not swayed by community pressure, who can recognize and refute indoctrination and propaganda, who are self-directing individuals with initiative and high ideals.

In addition to programming general mathematics courses for slow groups, we should, also, program difficult mathematics topics for the bright groups. The bright pupil should be stimulated with work which will tax his ability. There is no reason why such a pupil should be deprived of the use of the powerful tool of the calculus, of the knowledge of analytics, of the use of the planimeter, of practical surveying. As Dr. Thompson remarked in *Calculus Made Easy*, "What one fool can do, another can."

The mathematics club can offer such opportunities to the bright pupil. The use of the slide rule and its theory as a practical application of the use of logarithms, determinants in solving sets of three equations, elementary number theory concerning divisibility of numbers, graphical solution of maximum and minimum problems, the use of differentiation in maximum and minimum problems, the use of integration in area and pressure problems can be considered in club meetings. Casting out nines, magic squares, geometric fallacies, mathematical puzzles, the construction of Eratosthenes' sieve for primes are too elementary for serious mathematics students. They should be considered in the junior mathematics club. The seniors can follow with profit and interest L. Hogben's *Mathematics for the Millions* and M. Logsdon's *A Mathematician Explains*.

Mathematical contests are, also, valuable in developing exceptional mathematical ability. Contests in the classroom, contests between classes and school contests are equally exciting. New York City has an Interscholastic Algebra League with many schools competing. The national mathematical society, Pi Mu Epsilon, conducts contests for high school pupils. Recently I noticed an announcement concerning the results of the intercollegiate mathematical contest sponsored by the Mathematical Association of America. The cash prizes are attractively large. One of the winners for Massachusetts Institute of Technology was a boy who first started winning mathematical prizes and contests in this high school. His interest in mathematics was aroused through extra assignments of calculus problems in the algebra class, through the presentation of special reports before the mathematics club and through Pi Mu Epsilon contests. The school is proud of this particular graduate.

The small high school can further mathematical interest and develop leaders of thought through making available to pupils magazines, such as, THE MATHEMATICS TEACHER and *School Science and Mathematics;* a mathematics library in the classroom, containing books of the type of D. E. Smith's *History of Mathematics*,

L. Conant's *Number Concept*, W. W. R. Ball's *Mathematical Recreations and Essays*, G. Chrystal's *Algebra*, N. Altshiller-Court's *College Geometry*, and C. Smith's *Conic Sections;* a mathematics scrapbook compiled as a class project, containing puzzles, problems, letter divisions, mazes, fallacies, graphs, practical applications of mathematics, and mathematical news items; a collection of photographs forming a frieze above the blackboard, illustrating mathematics through the ages, mathematics in art, engineering, finance, astronomy, science, navigation, industry, business, architecture and aviation; a bulletin board covered with timely mathematical facts and figures; the brochures published by the American Council on Education and prepared by the Committee on Materials of Instruction on *The Story of Numbers, The Story of Weights and Measures, The Story of Our Calendar,* and *Telling Time throughout the Centuries;* and the monograph sponsored by THE MATHEMATICS TEACHER on *Numbers and Numerals* by D. E. Smith and J. Ginsburg. The small school should have an alert, lively mathematics club. It should sponsor mathematical contests as well as debating or athletic contests. By means of such procedures it will answer the challenge of the bright pupil. It will be truly democratic. It will graduate young men and women who are intelligent and thoughtful. It will train wise, competent and just leaders, capable of reasoning and of solving the difficult problems confronting our nation. It will offer an education which will be truly education in defense of democracy.

What Is Topology?

By Francis C. Hall, *New York, New York*
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A good part of the study of geometry in high schools hinges on two ideas: that of congruence of lines and that of congruence of angles. In the study of projective geometry, a basic idea is that of perspectivity. For example, the points $A_1A_2A_3$ are said to be "perspectively related" to

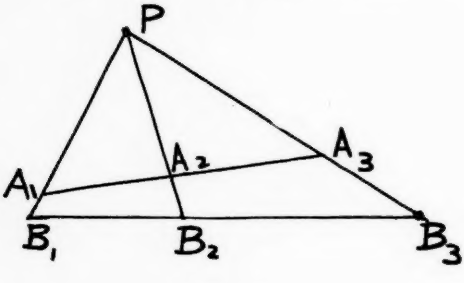


Fig. 1

$B_1B_2B_3$ if A_1B_1, A_2B_2, A_3B_3 meet in a common point P. (Fig. 1)

In topology, a study is made of properties of geometric figures that remain unchanged when certain kinds of transformations are performed on these figures. These transformations are called "homeomorphisms."

Consider the line segment AB and the curved line $A'B'$, (see Fig. 2). If we agree to match A with A', B with B', and if to

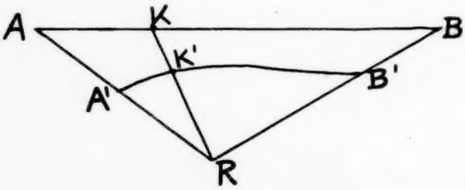


Fig. 2

every point on AB, for example K, we match a point of $A'B'$, for example K', in such a way that points very near K are matched with points very near K', then AB and $A'B'$ are said to be homeomorphically related. This is done in the adjoining figure by joining AA', BB' and using the point R in a way quite similar to the way P was employed in Fig. 1.

Consider two circles with radii r and R, placed concentrically. To every point A match the point A' on the same line through O and to every point P on OA let P' on OA' be matched where P is determined by the proportion $OP:R\,OP':r$. This produces a homeomorphism between the small and large circles. It is a two-dimensional relationship.

A homeomorphism is a bi-continuous and reciprocally one-to-one transformation. When we say that a transformation is continuous, we mean that "limit points" go into "limit points." In Fig. 3, for in-

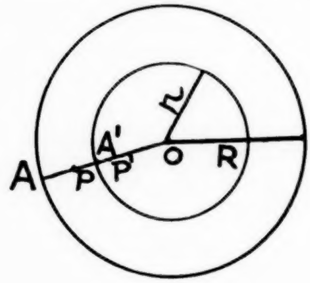


Fig. 3

stance as $P \to A$, $P' \to A'$ (the arrow means "approaches"). The word "reciprocally" means that as $P' \to A'$, $P \to A$, when applied to continuity. (Bi-continuous means the same thing as reciprocally continuous.) Similarly, "reciprocally one-to-one" means that for each P there is a P' and for each P' there is a corresponding point P.

There is a similar homeomorphism between points of a triangle and points of a circle. Place the triangle inside the circle. By what has gone before, we may assume a circle of any convenient size. We shall, moreover, assume that the center of the circle is inside the triangle. Let OR be any radius cutting ABC in R'. If P and P' lie on OR and if $OR:OP = OR':OP'$, then to every point P within the circle, there is a

point of the triangle, and conversely. Moreover, by this correspondence, the points R and R' are matched, and O is said to be self-corresponding, or an invariant point of the correspondence.

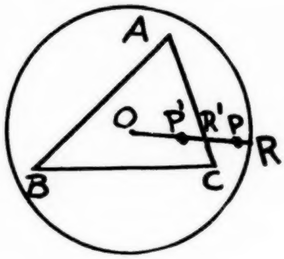


FIG. 4

Any two triangles can be made to correspond homeomorphically. This can easily be done by means of a system of so-called "barycentric coordinates," but we shall not go into details here.

A procedure analogous to that for the circle and the triangle would enable us to show that a sphere and a cube can be made to correspond homeomorphically.

If a rubber tire is cut along ABC and then along $ADEF$ and the rubber sheet

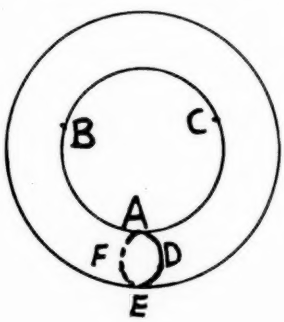


FIG. 5

resulting therefrom is stretched into the position shown herewith, the original surface is seen to be homeomorphic with the rectangle, if we consider all four vertices as the same point and if the opposite sides also be identified. The idea of considering several points of a figure as one and the same point is a very useful one in topology.

The subject matter of topology is so extensive that two so-called "schools" have grown up. There is the point-set-theoretic school, which is able to deal with spaces of the most abstract kinds imaginable. The other is the so-called combinatorial school. The latter deals with figures thought of as divided into simplexes. For example, a zero-simplex (0-simplex) is a mere point, a 1-simplex is a line segment, a 2-simplex is a triangle, a 3-simplex is a tetrahedron. To be explicit a topologist distinguishes between open and closed simplexes. "Open" implies a set of points exclusive of the boundary; "closed" implies the latter is to be included. Thus an open 1-simplex is a line segment without its end-points.

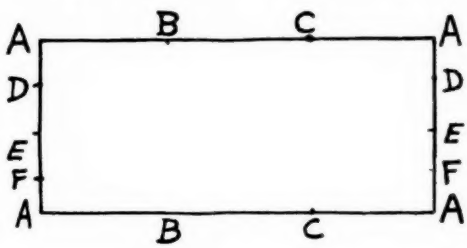


FIG. 6

The simplexes form so-called complexes. A complex is a set of points such that

(a) to every point there is at least one simplex,

(b) to every point there belongs only a finite number of simplexes,

(c) two simplexes are either without a common point, or they have a common "side," (this term is used here in a very broad sense to include vertices, faces and "sides" in the usual sense), or one is a "side" of the other.

The process of dividing into 2-simplexes or triangles in plane topology is generally known as triangulation. This may be done in an unlimited number of ways. In Fig. 6 we may, for example, join each pair of opposite A's, making four triangles. There are then two 0-simplexes, namely A and G, six 1-simplexes, namely a, b, c, d, e, f and four 2-simplexes, namely I, II, III, IV. This is shown in detail in Fig. 7.

The Euler-Poincaré characteristic is $N = -x_0 + x_1 - x_2$, where x_0 = the number

of 0-simplexes, x_1 = the number of 1-simplexes and x_2 = the number of 2-simplexes. This number N is an example of a topo-

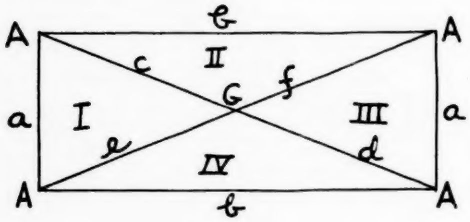


FIG. 7

logical invariant. Using the above mentioned figures, we find for the torus (the topological word for the rubber tire), $N=-2+6-4=0$.

If any other method of subdivision were used, as for example the one shown in Fig. 8, the value of N would be unchanged. Here we find $x_0=4$, $x_1=12$, $x_2=8$ and again $N=0$.

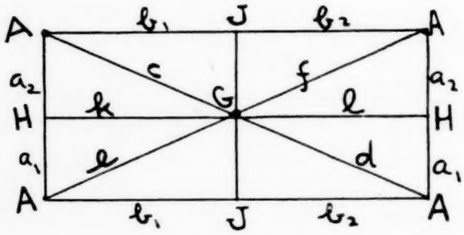


FIG. 8

In the case of the surface of a tetrahedron $x_0=4$ (there are four vertices), $x_1=6$ (six sides) and $x_2=4$ (four faces). Here $N=-4+6-4=-2$. For the surface of a cube $x_0=8$, $x_1=12$, $x_2=6$ and $N=-2$. Moreover, it can readily be shown that the surfaces of a tetrahedron and of a cube can be made to correspond hormoemorphically.

The surfaces mentioned are all known as "orientable." By this we mean that if a small curved arrow is moved around in any manner, it will return to its starting point with its direction unchanged. Every surface comes under one of two classes: it is either orientable or non-orientable. A surface in one class cannot be homeomorphic with one in the other.

If a strip of paper in the shape of a rectangle be twisted, and the ends glued together, a so-called Möbius strip is produced. (See Fig. 9.)

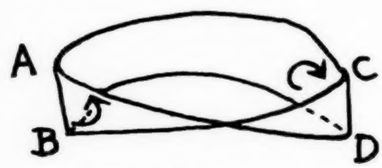


FIG. 9

A small curved arrow if moved around the surface returns to its starting place with its direction reversed. This is a classic example of a non-orientable surface.

We have touched on a small part of surface (two-dimensional) topology. The subject generalized to n-dimensional space is one that has received much attention within the last decade.

Converses of Algebraic Problems

By JOSEPH A. NYBERG, *Hyde Park High School, Chicago*

A FARMER *mixes 40 bu. of wheat worth 90c. a bushel with 60 bu. of corn worth 50c. a bushel to make some chicken feed. How many bushels of feed will he get and what is it worth per bushel?* (In the usual manner, we disregard the work and expense of grinding and the shrinkage in volume.)

This problem involves only arithmetic but a study of it is useful in the algebra class. The problem can be used to illustrate dependence since the value of the mixture per bushel depends on the number of bushels of each kind of grain and the respective values per bushel. Or, if the teacher likes to talk about functions, the value per bushel of the mixture is a function of certain variables. From the arithmetic problem the pupil can derive the relations

value of one grain + value of other grain = value of mixture

value of one grain = number of bushels × value per bushel

amount of one grain + amount of other grain = amount of mixture

These are the relations which the pupil must have in mind when he attempts the related algebraic problems.

There is nothing new in the preceding remarks. The use of arithmetic problems as an introduction to the related algebraic problems is common. But teachers may overlook certain other possibilities which these arithmetic problems offer; namely, the significance of "reversing" a problem. Neither the journals nor the textbooks on pedagogy discuss this topic. It can be explained as follows.

When the class has solved the problem above, the teacher says, "Suppose I tell you the answer to a problem of this kind; can you tell me the data from which I started? Suppose I tell you the value of the mixture per bushel; can you tell me how many bushels of each kind were used in the mixing?"

Undoubtedly the class would not even know what the teacher is hinting at if this is the first time that the teacher talks about "reversing the problem." I have used the problem about wheat and corn because it is a good example of a problem containing many, in fact six, variables. In a class the "reversing" attitude must be started earlier with simpler problems.

For example, at the end of the eighth grade the pupil has solved the three percentage problems. When he begins his work in algebra he learns that all three types of problems can be solved by writing the problem as an equation involving three numbers, one of which is unknown. Here he should learn not merely how to solve the problem but also learn the general principle:

If three numbers have the relation $ab = c$, and if two are known, then the third one can be found.

This is about the earliest the teacher can say, "If I tell you a and b, you can find c. But let us reverse the problem. If I tell you c, can you find a and b?"

Shortly afterwards the pupil solves the arithmetic problem:

If a dealer buys shoes for \$4 a pair and sells them for \$5 a pair, what is the profit and what is the per cent of profit based on the cost?

After this problem has been solved the teacher can say, "I have told you the cost and the selling price, and you have found the profit and the rate of profit. But let us now reverse the problem. If I tell you the profit and the rate of profit can you find the cost and the selling price?"

To see how rich this problem is in opportunities for teaching the nature of algebra let us consider it more in detail. The problem involves four numbers, c, s, p, and r, respectively, the cost, selling price, profit, and rate of profit. In the previous grades the problems assign values to c and s, and

the pupil finds p and r. In the ninth grade we are ready to ask such questions as:

If one of the four numbers is given, can you find the other three?

If two of the numbers are given, can you find the other two? In how many different ways can you select two of the numbers? Can you make a sensible problem for each choice?

If three of the numbers are given, can you find the other one? If three are given, can you assign any values you choose to these three, or must you be careful how you select them?

Such discussion will develop the general principle: No matter what the data is, there is one general method for finding the conclusion; namely, substitute the data in the relations

$$c+p=s,\ rc=p, \text{ and } c+rc=s$$

and solve the equations.

In the next stage, the pupil may be attacking *coin* problems. The first problem would be one that involves only arithmetic, such as

I have 7 dimes and 4 nickels. How many coins have I and what is their total value?

Then the teacher says, "Let us now reverse the problem. I shall tell you how many coins I have and their total value. Can you tell me how many of each kind I have?"

A perfect pupil would answer: First we must be conscious of what numbers are involved in the problem, and what the relations between the numbers are. Using n_1 and n_2 for the number of coins and v_1 and v_2 for the respective values of one of each of the coins, we have the relations

$$n_1+n_2=\text{total number of coins}$$
$$n_1v_1+n_2v_2=\text{total value.}$$

By substituting the data in these relations we can solve the reversed problem.

Naturally no pupil would use such language; but I have used it here as a means of pointing out that the problem in arithmetic can be used to teach the pupil to see what numbers are involved in the problem and what the relations are. Soon the pupils will get the idea that one of the main objects of algebra is that of solving the reversed problem. Arithmetic is sufficient for the direct problem, but algebra is needed for the reversed problem. This is one of the views of algebra which pupils should grasp. Finding square roots is the reverse problem of squaring; finding the equation of a graph is the reverse of drawing a graph from the equation. Although mathematicians speak of these problems as *inverses*, the word *reversing* is more suitable for a beginner's class in algebra. Further, the word *inverse* will be confusing when the pupil studies inverses of propositions in the geometry class. In the title of this article I have chosen the word *converse*, which can be justified as follows:

In the first problem in this article, the variables are

w, the number of bushels of wheat
c, the number of bushels of corn
a, the value of wheat per bushel
b, the value of corn per bushel
f, the number of bushels of mixture
d, the value of feed per bushel

Let us use the word *data* to mean the given numbers, and the word *conclusion* to mean the numbers which we seek. Then in the arithmetic problem the data is w, a, c, and b, and the conclusion is f and d. The definition of converse given by Lazar in his "Logic in Geometry" is: *the converse of a theorem may be obtained by interchanging any number of conclusions with an equal number of hypotheses.* Hence a converse of the arithmetic problem would have a, b, f, and d in the data, and w and c in the conclusion. This is the usual problem studied in the algebra class.

A further study of these ideas is recommended to students of pedagogy. For example: How should the study of time-rate-distance problems be introduced? What is a simple introductory problem, whose solution involves only arithmetic, and which lends itself naturally to the notion of reversing? Here the trouble is that most of the time-rate-distance prob-

lems in the algebra texts can be solved by arithmetic alone. The student might also investigate the problems in textbooks to see which types lend themselves naturally to the reversing process and which do not. Perhaps the pedagogic value of problems can be determined by the manner in which they can be thus treated. Does not this treatment also require an earlier treatment of sets of equations? What is the possibility of using this treatment of variables as a unifying concept? Too often the pupil learns to solve distinct types of problems without noticing that they are merely simple illustrations of attempts to find a conclusion from some data.

Mathematical Skills of College Freshmen in Topics Prerequisite to Trigonometry

By Jack Wolfe

Brooklyn College and School of Education, New York University
New York

The lack of retention of subject matter is an indisputable phenomenon which must be recognized in any program of education which is to be not merely consistent within itself and "logically" sound but also applicable in the world in which we live. The assumption that the students had at one time actually mastered the prerequisite topics and have retained their skills may lead to a well knit system of education on paper, but the failure of this assumption in practice may well be a chief factor contributing to some of our educational ills, especially those of student-failure and its consequent problems.

In support of the hypothesis that practical disregard of the actual knowledges, skills and abilities of our students as they come to us is a source of diminished efficiency of our teaching, we find that the subject most strongly of a sequitur nature, mathematics, has the greatest failing rate of all subjects in elementary school,[1] in high school[2] and in college[3] according to various reports. And the second ranking subject of a sequitur nature, foreign languages, has the second highest failing rate in high school[4] and in college.[5]

[1] R. L. Morton, *Teaching Arithmetic in the Intermediate Grades.* New York: Silver, Burdett Co., 1938, p. 7.

[2] New York City, Thirty-Ninth Annual Report of the Superintendent of Schools, 1936–1937, pp. 259, 260. Fortieth Annual Report, 1937–1938, pp. 231, 232. Forty-First Annual Report, 1938–1939, as reported in the *New York Times*, Sunday, Nov. 3, 1940, Education News Section, p. LD7.

[3] "Among 41,000 freshmen in 726 higher institutions of learning, the largest percentage of failure was reported in mathematics, the second largest percentage in Spanish," . . . R. Strang, *Personal Development and Guidance in College and Secondary School.* New York: Harper and Brothers, 1934, p. 167.

[4] Superintendent of Schools, *loc. cit.*

[5] R. Strang, *loc. cit.*

Undoubtedly there remain inherent difficulties of the subject itself even for adequately prepared students. But regard for the students' welfare demands a constructive treatment of the problem of background deficiencies and not a "passing-the-blame" dismissal of it.

The following table indicates that for a significant number of students the incidental classroom treatment of prerequisite topics in the trigonometry course did not suffice to correct background deficiencies. The first test was given to 257 students, the second to 280 and the third to 257. The three tests were identical.

It is verified from the analysis of the types of errors that there is no one concept which, if developed in the student, will eliminate all errors. The errors are varied and are specific; the remedial work should be designed to remove the specific defects. Mathematical ability includes the sum of many specific mathematical skills. Although interrelationships are present, it is not within the capabilities of most of our students to comprehend and apply them without specific direction. Specific remedial instruction is not to be conceived as treating each topic or skill as if it were entirely independent of all others, but rather as a method of indicating explicitly the relevant transfer and relationships.

Remedial Work

As a result of conducting with college freshmen an experiment in remedial work in the skills prerequisite to trignonometry, the writer believes that the most efficient way, administratively and educationally, that the institution can meet the problem of review is by the introduction of a definite, specific and individualized remedial program for the students who need it.

TABLE I

Performance on Each Item on Test on Prerequisite Skills

Topic and Items	Per cent Wrong: Near the beginning of the trigonometry course	Per cent Wrong: Near the end of the trigonometry course	Per cent Wrong: Near the end of the college algebra course	Chief Errors and Comments
Division Involving Zero				
Simplify $\frac{0}{4}$	22	13	17	4; ∞; omissions. The response ∞, or impossible, was made by 1% on the first test, 5% on the second, and 10% on the third.
Removing Parentheses				
Simplify $3\left(\frac{1}{3}+2\right)$	27	14	12	$\cancel{3}\left(\frac{1}{\cancel{3}}+2\right)=3$
Simplify $(4-2)(3-5)$	16	8	5	Signs.
Simplify $3(x-2y)-4(x-y)+2x$	21	15	19	Signs, $-4(x-y)=-4x-4y$
Multiply $(3x+5)(2x-6)$	18	15	9	"Reducing" $6x^2-8x-30$ to $3x^2-4x-15$
Decimals				Location of decimal point
Multiply, $(.2)(.3)$	10	6	4	.6
Divide, $\frac{.3}{.02}$	28	10	13	1.5, .15, .015, .0015
Divide, $\frac{.015}{.6}$	44	29	28	.25, omission of zero following the decimal point.
Arithmetic Fractions				
Simplify $\frac{2+3}{2+9}$	11	8	11	Cancellation of terms.
Simplify $\frac{4}{5}\cdot\frac{12}{13}-\frac{3}{5}\cdot\frac{5}{13}$	49	20	25	33 } Multiplied by denominator to "clear fractions"
Simplify $\frac{1}{2}\cdot\frac{3}{5}+\frac{3}{2}\cdot\frac{6}{5}$	39	20	18	21 }
Simplify $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-1}$	52	31	27	Greatly varied responses. Most common single error was $\frac{5}{3}$, error in sign.
Simplify (leave answer in radical form) $\frac{4+2\sqrt{3}}{2}$	51	41	33	Cancelling 2's with one term of numerator. Also changing $4+2\sqrt{3}$ to $6\sqrt{3}$.
Simplify $\sqrt{\frac{1-\frac{1}{2}}{2}}$	58	32	41	Changing $\frac{\frac{1}{2}}{2}$ to 1.
Arithmetic Radicals				
Simplify $\sqrt{16+9}$	19	8	9	7; treating $\sqrt{a+b}$ as $\sqrt{a}+\sqrt{b}$, or $\sqrt{a^2+b^2}$ as $a+b$. Order of operations.
Simplify $\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$	56	28	32	Omissions; greatly varied errors. Unsimplified form $\sqrt{\frac{1}{4}}$ and the erroneous response $\frac{1}{4}$ were the most frequent single errors.
Simplify $\sqrt{\frac{1-\frac{1}{2}}{2}}$	58	32	41	Changing $\frac{\frac{1}{2}}{2}$ to 1.
Simplify (do not change to decimals) $(\sqrt{5}+\sqrt{3})^2$	69	50	57	8; treating $(\sqrt{a}+\sqrt{b})^2$ as $a+b$ or $(a+b)^2$ as a^2+b^2. Order of operations.

TABLE I (*continued*)

Topic and Items	Per cent Wrong: Near the beginning of the trigonometry course	Per cent Wrong: Near the end of the trigonometry course	Per cent Wrong: Near the end of the college algebra course	Chief Errors and Comments
Approximating $\sqrt{n}$ as a Decimal				
Approximate $\sqrt{7}$ to the *nearest tenth*	58	32	43	2.7; probably due to changing 2.645 to 2.65 to 2.7.
Pythagorean Equation				
Solve (give positive answer only) $5^2+12^2=x^2$	11	5	6	17; taking $\sqrt{5^2+12^2}$ as $5+12$ Order of operations.
Solve (give positive answer only) $x^2+4^2=5^2$	9	8	5	1; taking $\sqrt{5^2-4^2}$ as $5-4$. Order of operations.
Solve (give positive answer only) $3^2+x^2=4^2$	25	17	14	$\sqrt{5}$, probably from $16-9=5$ a somewhat common error; perhaps from the 3, 4, 5 triad supplemented by the realization that 5 is too large.
Linear Equations				
Solve $\frac{3}{x}=\frac{9}{6}$	4	4	4	Varied errors.
Solve $\frac{x}{2}=\frac{4}{3}$	8	3	6	Varied errors.
Solve $2x+\frac{1}{2}=5$	23	21	20	$2\frac{1}{2}$, changing $2x+\frac{1}{2}=5$ to $4x=10$ as though the multiplication by 2 has "cancelled" the $\frac{1}{2}$. Also, 1, changing $2x+1=5$ to $4x+1=5$.
Solve for x, $(ab-1)x=ab-1$	36	14	12	Omissions; 0 by "cancellation."
Solve $4^2=3^2+5^2-2(3)(5)x$	32	26	21	Varied errors. Also changing $5x=3$ to $x=\frac{5}{3}$.
Factoring				
Factor x^2-4y^2	11	8	6	Omissions; and varied errors.
Factor $2a^2+a$	18	11	8	$2a(a+1)$; omissions; and varied errors.
Factor $3r^2-8r+5$	28	14	17	Omissions; and varied errors.
Quadratic Equations (other than Pythagorean Equation)				
Solve $x^2=3^2+4^2-2(3)(4)(\frac{7}{8})$ Give positive answer only	29	11	12	Varied responses. Chief single error was 4, the value of the right member.
Solve for all values of x, $x^2=9$	13	6	4	Only the plus answer despite the suggestive wording.
Solve for all values of x, $2x^2+x-6=0$	53	36	37	Omissions. Factors given as the solution. Errors in signs.

TABLE I (*continued*)

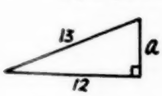
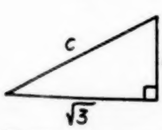
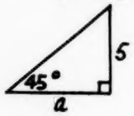
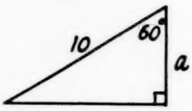
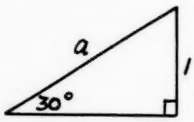
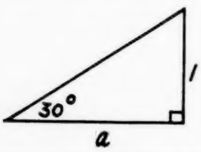
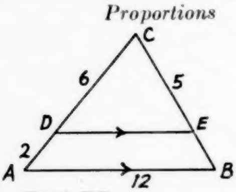
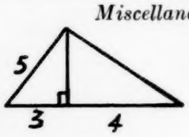
Topic and Items	Per cent Wrong			Chief Errors and Comments
	Near the beginning of the trigonometry course	Near the end of the trigonometry course	Near the end of the college algebra course	
Algebraic Fractions				Greatly varied errors.
Combine into one fraction $\frac{x+2}{x}+\frac{x+3}{3x}$ (The unsimplified form $\frac{4x^2+9x}{3x^2}$ was counted right)	42	20	26	$\frac{2x+5}{4x}$, sum of numerators over sum of denominators. $4x+9$, "multiplied through" to "clear fractions."
Simplify $\dfrac{\frac{a}{b}+\frac{b}{c}}{\frac{a}{b}-\frac{b}{c}}$	57	38	37	Greatly varied errors. 1, "cancellation." -1, "cancellation" or the belief that $x+y$ and $x-y$ are the negatives of each other.
Find the value of $\frac{2m}{1+m^2}$ when $m=\frac{3}{4}$	50	29	28	Greatly varied errors.
Combine into one fraction $\frac{5a}{a-2}-\frac{3}{2}$	62	41	49	Greatly varied errors. $\frac{7a-6}{2(a-2)}$, error in signs, $-3(a-2)$ to $-3a-6$. $7a+6$, "multiplied through" to "clear fractions."
Exponents				
Express as a power of x, $\frac{x^{10}}{x^2}$	39	19	6	x^5
Express as a power of x, $(x^2)^3$	45	35	23	x^5; x^8
Express as a power of x, $\sqrt{x^6}$	54	42	34	Omissions; x^4; $x^{1/6}$
Simplify 10^{-2}	55	35	36	Omissions; -100; $\frac{1}{10}$
Logarithms				
Given that $\log 2=0.3010$, find $\log \sqrt{2}$	68	23	37	Omissions; $\sqrt{.3010}$ or an approximation to it; .6020.
Given that $\log 3=0.4771$, find $\log 9$	77	27	42	1.4313; .14313
Pythagorean Theorem				
 Find a	22	4	10	Varied errors
 Find c	30	11	10	4, failure to extract square root of sum of squares.

TABLE I (*continued*)

Topic and Items	Per cent Wrong: Near the beginning of the trigonometry course	Near the end of the trigonometry course	Near the end of the college algebra course	Chief Errors and Comments
30° Right Triangle and 45° Right Triangle				
 Find a	14	8	6	Varied errors
 Find a	58	27	29	Omissions. 6, by confusion with 3, 4, 5 triangle. $5\sqrt{3}$, by association with wrong angle. $3\frac{1}{3}$, by imputing to sides the same ratio as their opposite angles.
 Find a	57	24	25	Omissions. $\sqrt{5}$, by taking second leg to be 2. 3, by imputing to sides the same ratio as their opposite angles.
 Find a	63	25	35	Omissions. 2, by imputing to sides the same ratio as their opposite angles.
Proportions 				Omissions. 2; 3
Find EB	47	28	47	
Find DE	60	48	59	Omissions. 10, from $\frac{DE}{12}=\frac{5}{6}$. 4, from $\frac{DE}{12}=\frac{2}{6}$. Wrong selection of the proportional parts.
Angle Measure How large in degrees is the angle formed by the hands of a clock at 2:00 o'clock?	32	18	23	30°; 45°.
Miscellaneous  Find the area of the triangle	57	39	61	Omissions. 28, failure to take one-half the product of base and altitude. 24, multiplication of segments of base. 48, multiplication of segments of base, and finally failure to take one-half the product of base and altitude.
Mean per item	37	21	23	

In the investigation the remedial group and the control group were formed as subgroups from the students who scored below-average on the initial test on topics prerequisite to trigonometry. They were virtually equivalent in each of the following characteristics:

1. score on the initial test on prerequisite topics,
2. score on psychological placement test,
3. chronological age, and
4. lapse of time since completion of last preceding mathematics course.

There were approximately sixty students in each subgroup at the beginning of the experiment and fifty at the end. The experimental group received approximately fourteen hours of remedial work devoted solely to the prerequisite topics while the other group received no special consideration.

The remedial teaching was concerned with developing meanings, and generalizations were usually arrived at inductively rather than by mathematically superior deductive proofs. Rote memorization and mechanized performance were discouraged.

The remedial syllabus upon which the review work was based consisted of the following topics. In each the degree of complexity was limited to that which the student would actually meet in his work of the trigonometry course.

Arithmetic

(a) Operations with zero
(b) Fractions
(c) Decimals
(d) Radicals
(e) Approximating $\sqrt{n}$ as a decimal

Algebra

(f) Removing parentheses
(g) Solving linear equation
(h) Solving Pythagorean equation
(i) Factoring
    (1) common factor
    (2) difference of two squares
    (3) trinomial in quadratic form, by cross-product method
(j) Solving equation of second degree
(k) Fractions
(l) Exponents
(m) Logarithms

Geometry

(n) Pythagorean theorem
(o) 30° and 45° right triangles
(p) Similar triangles (setting up the proportion and solving for one unknown)

Table II serves for a comparison of the control group, the remedial group and the total group of students of all levels of ability before the remedial work and after.

Table II

Comparative Growth with Regard to Score on Test on Prerequisite Skills

Time of Testing	Mean Per Cent Right Per Item		
	Control Group	Remedial Group	Total Group
Near the Beginning of the Trigonometry Course	51	50	63
Near the End of the Trigonometry Course	72	90	79
Near the End of the College Algebra Course	74	89	77

After the remedial aid, therefore, the experimental group performed better in the prerequisite topics than not only the control group but also the originally superior total group. More specifically, at the end of the trigonometry course the remedial group had attained greater skill in the prerequisite topics than even the students of the total group who received *A* as their final grade, while the corresponding performance of the control group was the same as that of the *D* students. At the end of the college algebra course, during which term no remedial assistance was given, the experimental group had again showed its superiority in these prerequisite skills over the students who received the final grade of *A* in college algebra, while the corresponding performance of the control group was between the levels of the *D* and the *C* students. Whatever weaknesses of trigonometry or college algebra may have

existed in the experimental group after the remedial program could not be attributed to persistent deficiencies in background skills.

Table III indicates the passing rates of the three groups with regard to final grades in trigonometry and in college algebra.

Table III

Success in Passing Trigonometry and College Algebra

Course Passed	Control Group	Remedial Group	Total Group
Trigonometry	85%	98%	91%
College Algebra	90	96	92
Both Courses in First Attempt	77	94	84

Thus it is seen that the remedial group attained a passing rate of ninety-four per cent for the year of freshman mathematics while the control group showed a corresponding passing rate of seventy-seven per cent. The difference between the proportions was 2.6 times the standard deviation of the difference.

As an additional follow-up phase of the experiment subsequent investigation revealed that seven per cent of the control group took and completed the first non-required mathematics course, analytic geometry, while nineteen per cent of the experimental group did so. Nor did the latter have any advantage in the number of science majors; for thirty-one per cent of the control group and twenty-seven per cent of the experimental group were so classified.

The evidence indicates that the remedial program did not "baby" the students so that they became dependent upon the continuation of such aid in their future work. On the contrary it appeared to have laid a superior foundation which outlived the immediate period of assistance and which enabled the students to sustain themselves later independently and better than if they had not received the earlier review.

"Would it be worthwhile for a university to set aside a brief period for individual diagnosis and remedial work, or is it better to begin new work on the false assumption that the students know the fundamentals?"[6]

[6] R. Schorling, "The Need for Being Definite with Respect to Achievement Standards," The Mathematics Teacher, XXIV (May, 1931), p. 320.

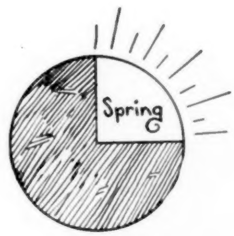
Spring as the Statistician Sees It

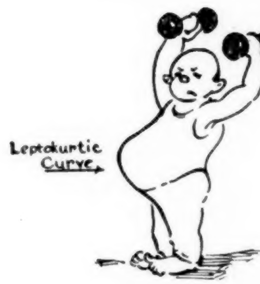
Words by T. H. TYLER, *King College, Bristol, Tennessee*

Drawings by HAZEL LUTHENA BEACH, *Graduate Student George Peabody College, Nashville, Tennessee*

1

Ah Spring, sweet quadrant of the year!
Kind welcomed variation
That brings from Winter's zero rank
A mild attenuation.





2

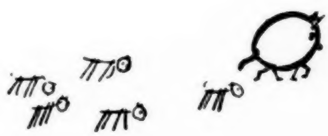
We leptokurtics exercise
And strive for restitution
Of platykurtic curvature
Or normal distribution.

3

Tall ogives* bloom in half our yards
With colors gay and nifty
(The semi-interquartile range
Approximates point fifty).

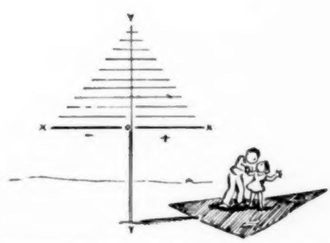
4

Huge quartiles chortle o'er the range
And 'long with each comes trooping
Her small percentiles, happy in
Their homogeneous grouping.



5

The trees their long abscissas stretch
To left and right of zero
And form a shaded area for
Our heroine and hero.



* Now all you sharks don't check too close
This springtime meditation
Wherein poor meter and poor math
Stand high in correlation;

For if this problem seems confused
Disorganized and muddy,
That simply goes to prove its worth
And need of further study.

6

The horizon's rough sketch he scans
  Till he her image spies
And there observes a set of curves
  More pleasing to the eyes.

7

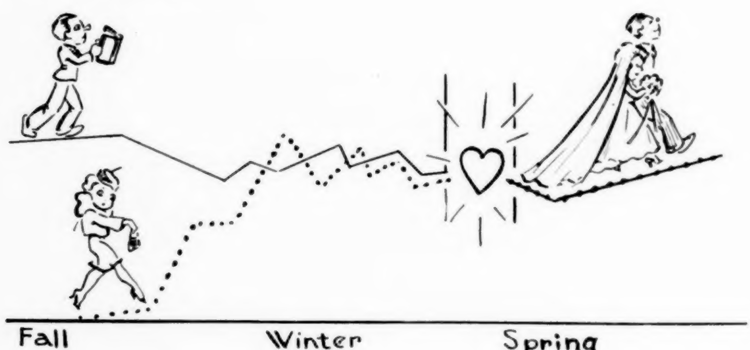
They tread an asymmetric path
  And now and then glimpse heaven
As kisses hourly they exchange
  (A mean of two-point-seven).

8

The dainty gamma nods its head,
  Likewise the flowering beta,
As he to her doth soft relate
  Ungrouped and spurious data.

9

And what results accrue from this?
  It leads to that condition
In which the plots of both their lives
  Find superimposition.



10

Thus Spring doth bring uncounted charms
  Of infinite summation;
And yet they almost all fall in
  Three sigmas variation.

A Report of the Seventh December Meeting of the National Council of Teachers of Mathematics at Baton Rouge, Louisiana

By H. W. Charlesworth
East High School, Denver, Colorado

This convention was held jointly with the Mathematics Association of America and the American Mathematical Society on December 30, 1940–January 1, 1941. 189 registered and about 350 attended.

General Theme: The Relation Between Enriched Mathematical Experience and Enriched Community Experience

The general trend of the talks made at this convention impressed one with the idea that mathematics is now, as never before, of greater importance. More and better mathematics must be taught in secondary schools. Although, in many sections of the country, certain courses in mathematics have been dropped out of the junior and senior high schools, there is good reason to believe that these will be put back and more along with them. It is very foolish to think that mathematics can be crowded out of the high schools in face of an ever increasing demand for it. So, the teachers of mathematics who attended this convention were made aware of the growing importance of their job.

Monday, 10:00 a.m.—Attendance: about 180

General Meeting

The convention got off to a good start in this section. F. L. Wren, George Peabody College for Teachers, Nashville, Tennessee, presided. Clarence A. Ives, Louisiana State University, Baton Rouge, gave the opening address on "What Should Enriched Community Experience Mean to the Secondary School Pupil?" Walter W. Hart of Winter Haven, Florida, spoke on "What Should Enriched Mathematical Experience Mean to the Secondary School Pupil?" This was followed by a talk by Etoile Wright of City Schools, Tulsa, Oklahoma, on "How May Teachers of Secondary School Mathematics Emphasize the Relationship Between Enriched Mathematical Experience and Enriched Community Experience?"

The titles of the talks given in this section are a fair summary in themselves. Mr. Hart advocated two sequences of courses in mathematics, one sequence of courses for those pupils headed toward engineering and technical fields, the other sequence of courses for those not capable of success in such formal mathematics or needing mathematics of a different type. Mr. Hart maintained that this second sequence of courses would be set up regardless of mathematics teachers, and that it is the job of the teachers of mathematics to supply new problem materials. The new social mathematics will be put in the 9th year, and algebra in the 10th year in smaller schools; in larger schools the two sequences should both start in the 9th grade, says Mr. Hart. The new enriched social mathematics sequence should be for those who are not capable or not inclined to do the formal sequence; the enriched algebra sequence should place more emphasis on understanding. Mr. Hart believes that much of the testing and drill done in mathematics teaching is deadening, especially when it is done on material that is already known. He contended that pupils should learn to *read*—their texts.

Miss Wright showed how various projects of home and family life could be used in junior high grades to show relationship of mathematical and community experi-

ences. She mentioned a number of practical problem situations that arise from the study of these projects. Certainly, mathematics need not die out because of lack of its practical every-day uses. Miss Wright concluded by saying that any teacher who desires to teach inspiringly can do so if she is willing to study and teach mathematics for what it is worth in everyday life.

MONDAY, 1:30 P.M.—Attendance: about 90

Mathematics for the Senior High School

A. R. Congdon of the University of Nebraska, Lincoln, presided. Elizabeth Dice, North Dallas High School, Dallas, Texas, spoke on "Word Problems in Algebra." We cannot judge what is usable in algebra by what one uses. We use only what we really know, said Miss Dice. Scientific men have now begun to recognize what they *can* use. Miss Dice suggested some practical word problems for use in algebra, and pointed out that the financial management of a home offers an unlimited number of practical problems. In daily life mathematics is being used, but not recognized as such. Miss Dice believes that students are far more interested in word problems collected from schools and daily life than they are in book problems. Selling campaigns offer excellent opportunities for practical problems. At least ten minutes each period should be allowed for understanding word problems. The substitution of meanings or symbols for words is good practice. Experimentation is necessary for the understanding of the formula. The inability of the old and young to follow rules is proof of the need of what algebra can teach. The slow-learning pupil may not be able to grasp what the upper fourth can master, but the slower group can at least learn to follow directions. If the slow-learners are to be *born,* the schools must cater to them. Formulas should be shown to classes illustrating how symbols are used. The physician writes a word problem for the pharmacist. At present word problems present a reading problem. Assigning a definite number of exercises in a definite unit is deadening to the class. Mastery of the processes involved is far more important.

Dorothy McCoy, Belhaven College, Jackson, Mississippi, spoke on "The Social Studies Curriculum and Mathematics." "Examine any daily newspaper edition and you will discover that the language of mathematics is not a dead one," declared Miss McCoy. Mathematics and the social studies can be mutually helpful. Mathematics makes a valuable contribution to the social sciences. The analysis of what constitutes a good citizen seems to demand a mathematical way of thinking. Would not well developed personalities require a knowledge of the principles of mathematics? Does not geometry furnish an ideal for critical thinking? Mathematics is utilized in our daily living; radio, charts, graphs, maps, tables, etc. Correlate history with the application and appreciation of mathematics. Teach concepts as well as methods. Cooperation in the mathematics class provides a real social situation where students live and work together. In the mathematics classes students may learn that the majority is not always right. The social science classes would do well to include the history of mathematics, the history of our economic systems and the history of our tools of thought. Such tools of thought as maps, projections, calendars, time, trigonometry (as it developed out of necessity), should be included. Modern man must learn science in self defense. The arithmetic of simple investments must be taught. The unbiased reasoning of mathematics is ideal training for citizenship. Those who do not know mathematics cannot appreciate its uses or possibilities.

"What Price Enrichment" was the title of the talk given by J. O. Hassler, University of Oklahoma, Norman, Oklahoma. According to Mr. Hassler, "The term, 'enrich,' has become a kind of Fifth Columnist in education." All too often this so called "enrichment" has really been a

"disenrichment." Too much time in mathematics courses is given to the non-mathematical or to exercises far below the grade and age level for which it is intended. Too much "socialization" of mathematics tends to kill the real value of mathematics and defeat the purpose it is intended to serve. If mathematics teachers must teach social science as well as mathematics, then we must keep mathematics intact and demand more time for the two jobs. Skilled workmen need to do simple problems that many of our high school students are unable to do. Propagandists of Progressive Education have attempted to prove that mathematics is non-essential. They overlook the fact that a good citizen needs a clear understanding of the fundamentals of mathematics in daily life. Tests given to thirteen hundred Oklahoma University freshmen showed them deficient in the mathematics needed for the average citizen. Under the Progressive Education setup the tendency is to crowd out too many of the fundamentals in order to "enrich" by bringing in the non-essentials.

Beware of any enrichment that does not contribute to logical thinking, problem solving, increased skills, and practical use, warns Mr. Hassler. We may relate our geometric thinking to the social sciences, but we must stick to the drill on the practical. We need motivation and enrichment, but the teaching of social science in arithmetic and mathematics courses at the sacrifice of the fundamental skills is not desirable.

MONDAY, 1:30 P.M.—Attendance: 34

Junior High School Section

H. C. Christofferson, Miami University, Oxford, Ohio, presided. E. G. Olds of Carnegie Institute of Technology, Pittsburgh, was introduced to read the paper of Nelson Tull of East Side Junior High School, Little Rock, Arkansas, on "The Logical Result of Enriched Pupil Experience."

Mr. Tull divided his subject into the two parts: (1) What is meant by enriched pupil experience? and (2) What is its logical result? Under the first of these he said, "We must lay emphasis on his individuality, provide for his originality and teach him to generalize his experiences."

Vergil Schult, Supervisor of Mathematics, Washington, D. C., then spoke on the subject, "Do We Practice What We Preach?" She quoted the professional philosophy of the Washington teachers as follows: "We believe that each child is an individual with certain needs which should be provided for in his experiences. We must help him to develop ideals of service, tolerance, increasing responsibility to his group and of the group to the individual." She then cited many types of problems considered in the classroom derived from actual community situations.

In the absence of Mr. Schorling of the University of Michigan, Ann Arbor, Miss Edith Woolsey of Minneapolis gave a talk on "Some Perplexing Problems Relating to Junior High School Mathematics." She introduced these questions: What are we going to do with children conditioned against mathematics in the lower grades? Is the solution to this problem to be found in building appreciations, in strong motivation, in homogeneous grouping, or in the history of the subject? Granted that every junior high school pupil, by the end of the seventh grade, should be able to add, subtract, multiply, and divide whole numbers, common fractions and decimals, what is our procedure to be with the many who do not know them? "How can the gap between elementary, junior and senior high mathematics better be bridged, by better teaching or by a more sequential course of study? Can one teach mathematics without teaching reading and correct speaking? Why bother with common fractions at all when it is so easy to teach decimals? How far shall we carry the subject of approximations? Do you find that the socialized approach to mathematics helps to solve the attitude toward it? Do the students get the mathematics they should have in this way? Is it worth while for the

teacher to spend so much time on materials for use in class as this method requires? Shall we teach mathematics by means of an activity or illustrate it with an activity?

These questions were thrown open for a discussion period in which Mr. Hart and Mr. Nichols spoke.

MONDAY, 1:30 P.M.—Attendance: 60

Teacher Training Section

H. W. Charlesworth, East High School, Denver, Colorado, presided in place of G. H. Jamison, State Teachers College of Kirksville, Mo., who was unable to attend.

Robert C. Yates of Louisiana State University read the paper of H. T. Karnes on "The Training of Southern Association Teachers of Secondary Mathematics" who was unable to attend because of illness. His paper was a report on an intensive study he had made on teacher preparation among secondary teachers in the Southern Association. The study revealed many interesting and striking facts. Mr. Karnes stated several conclusions resulting from the study. Although teachers of mathematics, those included in this study, are poorly prepared, much improvement has been made within the last ten years, says Mr. Karnes. Colleges do not require enough mathematics in teacher preparation. Present requirements are below standard, according to this study by Mr. Karnes. The teachers involved in this study recognize that they are poorly prepared and hold the mathematics departments at the universities at fault.

H. E. Buchanan, Tulane University, New Orleans, spoke on "The Role of Mathematics in Our Present Civilization." He began his talk with the statement, "That which is growing makes no noise." Of course, he was referring to mathematics and its persistent help in the development of a better civilization. Although the average citizen does not realize the value of mathematics to himself, it does affect the life of every man, says Mr. Buchanan. Relational thinking differentiates us from animals, he says. Thinking is very hard work and most people will not do it if they can get out of it. The study of mathematics *does* improve one's ability to think. Mr. Buchanan believes that much of the uncertainty regarding problems of national life is due to our not knowing how to think straight. Straight thinking is not to be thought of as merely a gift.

A good discussion followed.

MONDAY, 3:00 P.M.—Attendance: about 260

Mathematical Movies

Ruth Stokes, Winthrop College, Rock Hill, South Carolina, presided. Miss Stokes is a member of the Visual Education Committee appointed recently by the National Council. E. H. C. Hildebrandt, State Teachers College, Upper Montclair, N. J., is the chairman. Miss Stokes praised the work done by Mr. Hildebrandt and gave an explanation of the plans and accomplishments of the committee. She showed several reels of pictures relating to mathematics. Titles of some pictures shown follow: *Know Your Money, Geometry in Action, Rate of Change, Modes and Motors, Precisely So.* A seven-page mimeographed booklet was distributed. These listed several titles of motion pictures relating to mathematics, some silent and some sound, 16 mm. and 35 mm. films. These booklets also gave descriptions and reviews of several mathematical films.

While there have been some good films developed in the field of mathematics, it appears that much more can be done. This committee, no doubt, will be a stimulating factor for more and better films.*

MONDAY, 8:00 P.M.—Attendance: about 300

General Meeting

Although five meetings had preceded this one, it was at this time that J. O. Hassler, University of Oklahoma, intro-

* See Mr. Hildebrandt's article in the January 1941 issue of THE MATHEMATICS TEACHER.

duced S. T. Sanders, Head of the Department of Mathematics at Louisiana State University, who gave the address of welcome and Mary A. Potter, President of the National Council of Teachers of Mathematics, who responded. These two brief talks added much to the spirit of friendliness that marked the entire convention.

The address of the evening was given by W. B. Carver of Cornell University. In his introductory remarks, Mr. Carver asked that we face the issues before us squarely, because dogmatic statements get us nowhere. Our big problem has been, according to Mr. Carver, to teach mathematics for all and mathematics for the few in the same classes. He thinks that algebra should be taught to all in the ninth grade to serve as a basis of selection for those who are capable of continuing with the sequence of formal courses in mathematics. He stated that algebra could be useful to the average citizen, but the average citizen is *not* using it. To make algebra more useful, Mr. Carver suggests that we (1) put emphasis on its use, (2) deal with problems that have useful applications, (3) give a fuller meaning to *usefulness*, (4) find problems within range of experience and interest of pupils. Mr. Carver contends that mathematics is conducive to clear thinking and can be useful in helping pupils to do better thinking. He concluded his address by saying that mathematics must be taught for the million *and* for the chosen few.

TUESDAY, 8:30 A.M.—Attendance: 82

Mathematics for the Senior High School

F. L. Wren of Peabody College presided in the place of W. D. Reeve of Teachers College Columbia University, who was unable to attend.

E. G. Olds of Carnegie Institute of Technology, Pittsburgh, read a paper on "Use of Applications for Instructional Purposes"** in which he presented three questions: (1) Why use mathematical applications? (2) What applications should be used? (3) How use applications? Mr. Olds pointed out that applications are used to increase interest, aid in understanding, and to provide problems for practice. Mathematical applications can be found in the school subjects, in recreation, and in vocations. Applications should be used to enrich the course instead of depleting it. The student learns principles best by applying them. Mr. Olds gave numerous examples of good problem materials selected from various fields, problems within the understanding of pupils and concerning matters of interest to them. When choosing mathematical applications, consider the background and interests of children. Instead of considering the mathematics the average man uses, consider what he should be able to use.

A panel discussion followed. As Chairman of the panel, Mr. Olds introduced the members of the panel. Kate Bell of Lewis and Clark High School, Spokane, Washington, said that the region in which the pupil lives will supply material for mathematical applications. W. J. Borderlin, University High School, University of Louisiana, offered two assumptions: (1) We should teach that type of mathematics that can be applied in student living, (2) There are means of indicating the mathematical needs of pupils. Mrs. H. L. Garrett, Istrouma High School, University of Louisiana, gave several suggestions on the selection of problems. She advised that the teacher keep the history of mathematics in mind. Two of her pupils, a boy and a girl, assisted Mrs. Garrett in her discussion. H. S. Kaltenborn, Louisiana Polytechnic Institute, Ruston, Louisiana, stated that applications are not usually included in first year algebra, but they should be used. Problems of home building and financing furnish many applications. Thriza Mossman, Kansas State College, Manhattan, suggested that sometimes a concept is simpler than its application, and the selection of applications should be tempered with

** See the January 1941 issue of THE MATHEMATICS TEACHER for the complete paper.

careful reasoning. Frank A. Rickey of Louisiana State University said that curiosity satisfaction should be obtained from mathematical applications. Curiosity thrives on exercise and will die if not used. Mr. Rickey suggested several examples that interest pupils because they answer their questions about which they are curious. Applications do not have to be of the bread-and-butter type; it is sufficient if they satisfy the child's curiosity.

TUESDAY, 8:30 A.M.—Attendance: 30

Teacher Training Section

L. H. Whitcraft of Ball State Teachers College, Muncie, Indiana, presided.

Jessie Hoag of Jennings High School, Jennings, Louisiana, in her talk on "Help Needed by Teachers in Service" asked that supervisors give teachers time to exchange ideas. She asked that materials, books, magazines, and instruments be made available. Miss Hoag suggested that teachers need a bit of "prodding" to do their professional study, that training school courses should be taught by teachers who know, that part-time teaching is a hindrance to good mathematics teaching.

H. G. Ayre of Western Illinois State Teachers College, Macomb, Illinois, spoke on "The Place of the Supervisor in Training Mathematics Teachers in Service." He said that in-service training is most effective with young teachers and with progressive teachers of experience. Teacher training institutions should take the responsibility of helping to see that supervision is given to teachers as a part of their follow-up service. Mr. Ayre would hold the supervisor responsible for maintaining skills in arithmetic throughout grades 9 to 12. This, he says, should be done in new settings. Methods of teaching should be left to the freedom of the teacher, not dictated by the supervisor.

Mary Potter, Supervisor of Mathematics, Racine, Wisconsin, substituted for Raleigh Schorling of the University of Michigan, who was unable to be present. Miss Potter spoke on "Desirable Characteristics of a Supervisor of Mathematics." Miss Potter believes that excellent teachers do not necessarily make good supervisors. She outlined about twenty characteristics of a good supervisor and elaborated on some. To be a good supervisor one must be fond of children. Besides being an outstandingly good teacher and disciplinarian, a supervisor must be versatile, aggressive, systematic, a good organizer, tolerant, democratic and interested in human affairs. Miss Potter bebelieves a supervisor should do some teaching and should be a teacher by example.

An excellent discussion followed.

TUESDAY, 8:30 A.M.—Attendance: 35

Arithmetic Section

C. C. Ritchmeyer of Central State Teachers College, Mt. Pleasant, Michigan, presided.

H. C. Christofferson of Miami University spoke on "Useful Mathematics That Children Enjoy." Mr. Christofferson recommends using a specific problem to get a generalization as we use generalizations in geometry. Also, in problem solving, the important thing is not the answer, but the variable things that determine the answer. Work out a formula for problem solving as you would for finding the area of a circle. Relationships and formulas are more important to the child than mere problem solving.

Thelma Tew, Florida State College for Women, Tallahassee, Florida, spoke on "A Meaningful Way of Teaching Division of Fractions." Miss Tew said that children do not know the symbols or the language of arithmetic. The primary grades, perhaps, do the best job of teaching. When too much is crowded into the upper grades, the social application is lost. Considerable time should elapse before the introduction and the final conclusion in teaching skills. Do not burden one or two grades with too many skills. In teaching, preserve the na-

ture of the process so that the child may see the relationships. Do not teach something today that may be a hindrance tomorrow.

"Making Arithmetic of Greatest Value to the Community" was the title for the address given by R. L. O'Quinn of Louisiana State University. According to Mr. O'Quinn, "Knowledge becomes scientific as it becomes mathematical." Arithmetic is an important part of all mathematical structure. Arithmetic is a branch of mathematics that touches daily life, therefore the teacher should utilize the contacts of daily life. Mining, agriculture, local industry and wider fields will afford opportunities for community contacts. Talks with local people, a mathematical museum and mathematics clubs will furnish means of motivation: Making equipment for the mathematics classroom provides another means for stimulating interest. Arithmetic should not be considered as merely a tool subject. Applications as well as skills should be emphasized. Teach the student to think through a problem and to determine why certain skills are used. Arithmetic is a science as is any other type of mathematics. The thought processes in arithmetic are eternal. Teach the child not to accept or memorize rules before thinking the problem through. Make concrete applications. Organize materials that they may be taught in logical sequence and as the needs arise. Mr. O'Quinn believes that Arithmetical Analysis or a good course in mental arithmetic should be included in the high school curriculum. Those who would eliminate arithmetic, algebra and geometry are unaware of their values. No subject in the curriculum has more value than arithmetic, especially as an aid in studying other subjects. Both the direct and indirect results are valuable. Lack of adequate mathematics preparation closes many fields to the student. Today, more than ever before, mathematics is essential for the college student. The more complex curriculum of today demands more mathematics and a better foundation in arithmetic. All the foundational work for higher mathematics, essential for numerous fields of work today, begins with a thorough foundation in the fundamentals of arithmetic.

TUESDAY, 10:30 A.M.—Attendance: about 200

Multisensory Aids

Ruth Stokes, Winthrop College, Rock Hill, South Carolina, presided in the absence of E. H. C. Hildebrandt, State Teachers College, Upper Montclair, New Jersey.

Harriet Herbert of Edmunds High School, Sumter, South Carolina, gave a talk on "Fourth Dimensional Models in Secondary School Mathematics." Miss Herbert displayed models made by students from balsa wood, models to demonstrate the expansion of the binomial, the tesseract or hypercube, of course, to demonstrate $(2a+b)^4$. She told of her experiences with these figures and of the enjoyment she and her pupils received from their work with them. Miss Herbert has written a good article on this subject in a late issue of *The National Mathematics Magazine.*

Robert C. Yates of Louisiana State University talked on "Geometry via the Laboratory." Mr. Yates presented arguments in favor of teaching geometry by means of other instruments in addition to the straight edge and compasses. He displayed many models of linkages and paper cuttings, and showed how interesting the learning of geometric principles could be through the use of such devices. In addition to this display of devices in connection with his talk, Mr. Yates had arranged a very interesting and worthwhile mathematics exhibit in two classrooms in Nicholson Hall.

Kate Bell of Lewis and Clark High School, Spokane, talked briefly on "Visual Aids—Student Made." She presented slides showing the use of such materials for vitalizing mathematics teaching. Miss Bell recommended, of course, that students

have a real part in making these visual aids.

TUESDAY, 1:30 P.M.

Trip to St. Francisville

This tour was made available without cost to those attending the convention; more than a hundred made the trip. It included a visit to two ante-bellum estates, one which had not been reconstructed and the other that had been somewhat modernized. This trip caused the visitor to realize, to some extent, the spirit of the old plantation life and culture. Those who made the trip of approximately eighty-five miles felt that they had spent four hours very profitably.

TUESDAY, 7:00 P.M.—Attendance: 50

Southern University Meeting

This meeting was held at Southern University, which is located a few miles north of Baton Rouge. S. T. Sanders of Louisiana State University presided. Mary A. Potter, President of the National Council of Teachers of Mathematics, spoke on "The National Council of Teachers of Mathematics: Its Purpose and Program." H. C. Christofferson, Miami University, Oxford Ohio, substituted for W. D. Reeve, who was unable to be present. Mr. Christofferson spoke on "How Council Publications Serve Teachers of Mathematics." Ethel Harris Grubbs, Head of the Department of Mathematics, Divisions 10–13, Public Schools, Washington, D. C., spoke on "How the National Council of Teachers of Mathematics May Serve the Negro Teachers and How They May Serve the Council." Following this meeting the negro teachers at Southern University gave a reception in honor of Mrs. Grubbs.

WEDNESDAY, 8:30 A.M.—Attendance: 30

Arithmetic Section

Edith Woolsey of Minneapolis presided. F. L. Wren of Peabody Teachers College spoke in place of Raleigh Schorling of the University of Michigan. Mr. Wren spoke briefly about the work of the National Committee that is preparing a compendium of mathematical applications for the Sixteenth Yearbook.* Mr. Wren said that those who have been working on the committee believe that arithmetic instruction must be socially significant and mathematically meaningful. Keep in mind not what is used, but *what might be used* and *what ought to be used* in arithmetic, says Mr. Wren.

Incidental learning programs are inclined to be haphazard, too much varied by the pupils' suggestions and not enough teacher planning. The teacher should be able to recognize concepts, values and needs of arithmetical procedure. Teachers sometimes fail to recognize that arithmetic problems are all around us, that daily living offers good problem material. Using this material is the responsibility of the teacher. The textbook offers only a guide. Any community offers an abundance of good problem material.

J. O. Pettis, Louisiana State University, Baton Rouge, spoke on the subject, "How Instruction in Arithmetic Might be Improved." According to Mr. Pettis, "Social utility is now replacing formalized training." Tests in measuring brought about accuracy and speed with less emphasis on some other good features to be emphasized in the teaching of arithmetic. He believes that a continuous program of mathematics from grades one to eleven is desirable. Nearly all pupils of all levels can assimilate the essentials of mathematics. Mathematics cannot be taught incidentally and be taught successfully. The aims of instruction largely determine the values to be derived. Remove emphasis from drill to problem solving. The fundamental processes should be mastered, but drill should be practical. The teacher should understand the aims of the various grades. Emphasize meaningful problems. Arithmetic should be taught in such a way that the

* See the advertisement of this yearbook on the outside back cover of this issue.

thinking process will follow through high school. There is too little planned guidance given pupils in problem solving. We can expect no improvement in mathematics teaching until there is a very definite co-operation between administrators and teachers. Better arithmetic teaching will truly enrich the way of living.

W. L. Hart of the University of Minnesota spoke on "War Preparedness." Mr. Hart appeared on the program instead of C. Newton Stokes of Temple University, Philadelphia, who was unable to attend. At present, Mr. Hart is acting as Mathematics Consultant in the present defense program. He is working in the interest of qualified mathematicians whose skill may be turned to service of national defense and industries of national defense. His work is especially concerned with undergraduate mathematics on down to the lower levels, and especially secondary mathematics. Mathematics in industry, as well as in the defense program, is part of the study. The opportunities for workers trained in mathematics, in industry and in aviation is greater today than ever before. Industry puts extreme demands on mathematics. There now is, and will be for some time, a shortage of skilled engineers. It is now a recognized fact that computational trigonometry is quite essential for skilled workers. The following recommendations were offered by Mr. Hart: A study should be made to show up the weaknesses of our present program, goals for adjustments should be set, advertise the need for more mathematics in the secondary schools, teachers of mathematics should hold meetings to study the problem of mathematics as related to present day needs, advisors should advise capable students to take mathematics as a part of national service, socialized mathematics should be interpreted to mean mathematics for industry and defense, solid geometry should be brought back to its rightful place in the schools, high school students who have been so unfortunate as to have missed much of mathematics should be given at least an abbreviated course in grades eleven and twelve.

WEDNESDAY, 8:30 A.M.—Attendance: about 175

Joint Session of N. C. T. M. and M. A. A.

Marie J. Weiss, Sophie Newcomb College, New Orleans, presided. Virginia Modesitt of Wright Junior College, Chicago, spoke on "The Teaching of Mathematics in Junior College." Miss Modesitt stated that, although mathematics is not required at Wright, about one-third of the students are taking mathematics in any given semester. Those students differ widely in their achievement and ability in mathematics, therefore there is an attempt on the part of the mathematics department to furnish the type of mathematics needed by the individual. This is done by means of a testing program. The regular freshman course is Introductory Mathematical Analysis. Miss Modesitt feels that much can be done for all pupils relative to their needs in mathematics by the use of a program as is being used at Wright Junior College.

D. R. Curtiss of Northwestern University spoke next, on "The Professional Interests of Mathematics Instructors in Junior Colleges." Mr. Curtiss gave great credit to Miss Martha Hildebrandt for the compilation of the report which he gave. Miss Hildebrandt is chairman of the committee appointed by the Mathematical Association of America to make this particular study. The study showed the professional attainments of instructors of mathematics in junior colleges, and pointed out that they and related groups are not being sufficiently served by any national organization. The report recommended that the Mathematics Association of America do more, particularly through its publications, to help the mathematics teachers in junior colleges.

H. M. Cox, University of Nebraska, Lincoln, talked on "What Are the Admin-

istrative and Guidance Uses of Mathematics Examinations?" He advocated statewide testing programs for the purpose of evaluating pupils' achievements and ascertaining their aptitudes and abilities. He mentioned particularly the Co-operative Test Service of the American Council on Education. Mr. Cox pointed out that the right use of tests is of great value and that we, as teachers, should not overlook this fact.

WEDNESDAY, P.M.

Of course, many went to the Sugar Bowl game. The weather was fine and the distance not too great, New Orleans being about seventy-five miles from Baton Rouge. Others spent some time in looking around Baton Rouge, particularly the magnificent capitol building. Many used the afternoon as a time of respite and sociability.

At 7:30 the joint banquet with the Association and the Society was held at Foster Hall. Banquet speeches were limited to 10 minutes each and were given by Paul M. Herbert, President of L. S.U.; by H. E. Buchanan of Tulane; by C. V. Newson, University of New Mexico. G. C. Evans, University of California; and by Mary Potter. This closed another very successful and wonderful convention of the National Council of Teachers of Mathematics.

THE ART OF TEACHING

The Determinant Notation

By L. Leland Locke, *Brooklyn, New York*

A Lesson on Choosing a Pattern and on Adhering to that Pattern

The solution of an algebraic problem usually falls into a conventional form which becomes the model for future problems of the type. There is neither choice nor variation. Thus we speak of the mechanics of algebra. Success in the subject depends upon the ability to recognize a type and a mastery of the mechanics necessary to the solution.

In building the determinant notation a simple pattern is chosen in the case of two simultaneous equations and upon this pattern the whole structure of the determinant notation depends. The manipulation of determinants is up to this time a unique process in algebra and may be compared with the working out of a puzzle by trial and error. Of many valid procedures the student is to choose that particular one which will involve an economy of manipulation.

A Permutation is any arrangement of unlike elements. Natural order is the order originally chosen as such. It may be the order of the letters of the alphabet or it may be the order reversed, or any other preassigned order. An inversion occurs with any variation of the natural order and consists of any element preceding an element which it follows in the natural order.

There are six permutations of the first three letters of the alphabet. There are several ways of choosing these systematically. The most common is to write the last two elements first in their natural order and follow by inverting them, gradually moving the other elements from right to left until each element has occupied each place.

Permutations	Inversions			
abc	0			
acb	1	*c* before *b*		
bac	1	*b* before *a*		
bca	2	*c* before *a*	*b* before *a*	
cab	2	*c* before *a*	*c* before *b*	
cba	3	*c* before *b*	*c* before *a*	*b* before *a*

The Determinant Notation

Consider the pair of simultaneous equations

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

where the coefficients indicate columns and the subscripts indicate rows. The form $a_1b_2 - a_2b_1$ appears in the solution. Let this expression be written in square array. This may be done in several ways. Choose that one which conforms to the arrangement of the coefficients in the pair of equations.

$$a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

Note that the minus sign does not appear but is considered as belonging to the secondary diagonal, from lower left to upper right. Note the positions which the elements take in the square array, as the whole argument depends upon adherence to this convention.

$$(1)\,(2) - (3)\,(4) \quad \begin{vmatrix} (1) & (4) \\ (3) & (2) \end{vmatrix}$$

Consider the set of simultaneous equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Write the three letters *abc* six times.

$a\,b\,c \quad a\,b\,c \quad a\,b\,c \quad a\,b\,c \quad a\,b\,c \quad a\,b\,c$

Permute the subscripts and note the inversions above each term.

$$\text{(I)} \quad \begin{array}{cccc} 0 & 1 & 1 & 2 \\ a_1b_2c_3 & -a_1b_3c_2 & -a_2b_1c_3 & +a_2b_3c_1 \\ & 2 & 3 & \\ & +a_3b_1c_2 & -a_3b_2c_1 & \end{array}$$

If the number of inversions is odd assign the minus sign to the term, otherwise the plus sign as above.

Factor and collect.

$$a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$$

Write the expression in two order determinant form.

$$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

Build this expression into a three order determinant by noting letters as column indicators and subscripts as row indicators.

$$\begin{array}{ccc} a_1 & & \\ & b_2 & c_2 \\ & b_3 & c_3 \end{array}$$

The second term is easily located. Place the a_2 in first column, second row, then the b_1 and c_1 in the first row. The b_3 and c_3 are already in place.

The form now stands.

$$\begin{array}{ccc} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ & b_3 & c_3 \end{array}$$

The sign $-$ does not appear but is regarded as belonging to a_2

The only element of the third term to be placed is a_3

Placing this element and drawing the bars we have the three order determinant.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

In the development of (I) and in building the determinant, there is no rewriting, which is here necessary to show the steps in the process.

The determinant is formed by writing the coefficients of the three equations in the order in which they appear and drawing the two bars. Reversing the above process gives the development of a three order determinant according to the elements of the first column.

If

$$A_1=\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}, \quad -A_2=\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}, \quad A_3=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

the determinant may be written

$$a_1A_1+a_2A_2+a_3A_3$$

Again, write the three subscripts six times in natural order

1 2 3   1 2 3   1 2 3   1 2 3   1 2 3   1 2 3

Permute the letters and note the inversions above each term

$$\text{(II)} \quad \begin{array}{cccc} 0 & 1 & 1 & 2 \\ a_1b_2c_3 & -a_1c_2b_3 & -b_1a_2c_3 & +b_1c_2a_3 \\ & 2 & 3 & \\ & +c_1a_2b_3 & -c_1b_2a_3 & \end{array}$$

Signs are determined as above according to number of inversions.

Factor and collect.

$$a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$$

Write the expression in two order determinant form.

$$a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}$$

Build this expression into a three order determinant as before.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

This determinant may be written

$$a_1A_1+b_1B_1+c_1C_1$$

Comparing (I) and (II) it is seen that the two expressions are identical. Hence in a three order determinant rows may change to columns and columns to rows.

EDITORIALS

Meaningful Rather Than Mechanical Mathematics

The reason why arithmetic in particular and mathematics in general do not come up to expectations in the lives of people in this present age is that the fundamental ideas of the subject are not understood by those who study it as well as they should be. What we have done traditionally is to perfect some sort of technical procedure in order to facilitate these fundamental ideas in particular instances and as Whitehead[1] points out, "The unfortunate learner finds himself struggling to acquire a knowledge of a mass of details which are not illuminated by any general conception." He says further, "Without a doubt, technical facility is a first requisite for valuable mental activity: we shall fail to appreciate the rhythm of Milton, or the passion of Shelley, so long as we find it necessary to spell the words and are not quite certain of the form of the individual letters. In this sense there is no royal road to learning. But it is equally an error to confine attention to technical processes, excluding consideration of general ideas. Here lies the road to pedantry."

[1] Whitehead, A. N., "Introduction to Mathematics," p. 8. Henry Holt and Company, 1911. A book which every teacher of mathematics should read.

The keynote of the new arithmetic, according to the Sixteenth Yearbook of the National Council of Teachers of Mathematics,[2] is that it should be "meaningful rather than mechanical." Thoughtful teachers working on the secondary level well understand that better understandling and appreciation of the fundamenta-ideas of mathematics are the most important goals today. People attack mathematics mostly because of unfortunate experiences they have had with it, even to the extent of having failed in it. Or, even in the case of those who have received passing marks or better, many admit that they really saw little of value in the subject.

The time is certainly here when not only textbooks, but teaching practices as well, should be reorganized to bring about improved conditions. The Mathematics Teacher will be glad to have further contributions which can outline some better if not newer methods of procedure.

W. D. R.

[2] "Arithmetic in General Education," final report of the National Council Committee on Arithmetic. Bureau of Publications, Teachers College, 525 West 120th Street, New York, New York. Price $1.25 postpaid.

The Boston Meeting

The 1941 Summer Meeting of The National Council of Teachers of Mathematics will be held this year in the historic old City of Boston in connection with the annual meeting of the National Education Association. Here is a chance for teachers of mathematics to plan a most interesting New England vacation, see the historic sites in Boston and improve their professional outlook by joining their friends and fellow workers at the National Council Meeting. President Potter has promised a good program; so save the dates June 29th to July 3rd inclusive. The complete program will appear in the May issue of The Mathematics Teacher.

W. D. R.

◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
The Bronx High School of Science, New York City

The American Mathematical Monthly

December 1940, Vol. 47, No. 10

1. von Mises, Richard, "Mathematical Problems in Aviation" pp. 673–685.
2. Thomas, J. M., "The Resolvents of a Polynomial," pp. 686–694.
3. Bradley, A. D., "The Gnomonic Projection of the Sphere," pp. 694–699.
4. Arnold, W. C., "How to Study Mathematics," pp. 704–707. A portion of "The Guide to Study for De Pauw Freshmen."

January 1941, Vol. 48, No. 1.

1. Dorwart, H. L., "Comments on the North Carolina Program in Freshman Mathematics," pp. 37–39.
2. Keller, M. W., Shreve, D. R., and Remmers, H. H., "Diagnostic Testing Program in Purdue University," (continued) pp. 39–41.
3. Kennedy, E. C., "Concerning Nearly-equal Roots," pp. 42–43.
4. Whitman, E. A., "The Use of Models while Teaching Triple Integration," pp. 45–48.
5. Singer, James, "Some Remarks on Coordinate Systems," pp. 49–53.

Bulletin of the Association of Teachers of Mathematics of New York City

February 1941, Vol. 4, no. 3.

1. Committee on Approximate Computation, "Syllabus Recommendations for Grades 7–12," pp. 6–11.

Bulletin of the Kansas Association of Teachers of Mathematics

December, 1940, Vol. 15. no. 2

1. Price, G. Baley, "Report of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America," pp. 23–25.
2. Beito, Edwin A., "Mathematics of Aviation," pp. 25–28.
3. Hassler, J. O., Placement Test for Freshmen," pp. 28–29.
4. Betz, William, "The Problem of Non-College Preparatory Mathematics Viewed in the Light of a Broad Program of General Education," pp. 29–31.
5. Read, Cecil B., "Fun and Fact with Figures," pp. 32–34.

National Mathematics Magazine

December, 1940, Vol. 15, no. 3.

1. Pawley, Myron G., "New Criteria for Accuracy in Approximating Real Roots by the Newton-Raphson Method," pp. 111–120.
2. Finkel, Benjamin F., "A History of American Mathematical Journals" (continued), pp. 121–128.
3. Yates, Robert C., "The Trisection Problem" pp. 129–142.

This is the first in a series of five chapters. It deals with the problem under the following subheadings: (a) The famous three; (b) A classical game; (c) Trisection; (d) Statement of the problem; (e) Constructibility; (f) The impossible; (g) The possible; (h) Other criteria; (i) Regular polygons.

January, 1941, Vol. 15, no. 4.

1. Simmons, H. A. "Some Characteristics of a Good Review of an Elementary Mathematical Textbook," p. 162.
2. Finkel, Benjamin F., "A History of American Mathematical Journals" (continued), pp. 177–190.
3. Yates, Robert C., "The Trisection Problem," pp. 191–202.

This is the second in a series of five chapters. It deals with the solution of the trisection problem by means of the following curves: the quadratrix, the conchoid, the hyperbola, the limaçon, the parabola, the cubic parabola, and the cycloid of Ceva.

School Science and Mathematics

February 1941, Vol. 41, no. 2

1. Carnahan, Walter, H., "Some Desirable Curriculum Adjustments in Science and Mathematics," pp. 103–114. A paper panel with the following participating: Veva McAtee, Ira C. Davis, and Joseph A. Nyberg.
2. Thompson, R. B., "Diagnosis and Remedial Instruction in Mathematics," pp. 125–128.
3. Sadowsky, Michael A., "Does the Law $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$ Hold for Imaginary Numbers? Does It Hold at All?", pp. 128–130.
4. Kinney, Jacob M., "The Law of Tangents in Modified Form and Some Other Related Formulas," pp. 158–159.
5. Schaaf, William L., "Unexplored Possibilities of Instruction in Graphic Methods," pp. 160–171.

An extensive bibliography of nine pages is included.

6. Read, Cecil B., "A Note on Parametric Equations," pp. 178–179.

NEWS NOTES

The Cornell University Graduate School of Education will conduct a workshop at Ithaca for 6 weeks from July 7 to August 15, 1941 under the directorship of Professor M. L. Hulse.

The Workshop is designed for teachers in the elementary and secondary schools. The cooperating divisions are English, social studies, science, mathematics, home economics and agriculture. Members of the Education and Arts staff at Cornell will share with members of the Workshop the responsibility for developing projects relating to the teaching of democracy. This theme is stressed because teachers are under pressure to make a greater contribution toward the attainment of this goal. To insure a realistic approach the Cornell Workshop staff is surveying the needs of a representative central school in upstate New York. The faculty of this central school later will attend the Workshop to carry forward plans for reorganizing the school curriculum.

The fourth meeting of the Men's Mathematic's Club of Chicago was held on Friday Jan. 17, 1941 at the Central YMCA. The speaker was Anatol Rapoport whose topic was "Music and Mathematics."

Teachers College, Columbia University, will offer the following courses in the teaching of mathematics this summer, July 7 through August 15, 1941:

By Professor A. Day Bradley: Navigation, professionalized subject matter for teachers. Professor J. R. Clark: Teaching algebra in secondary schools; Teaching geometry in secondary schools. Dr. Nathan Lazar: Teaching algebra in junior high schools; Logic for teachers of mathematics. Mr. Gordon R. Mirick: Demonstration class in plane geometry; Elementary mechanics, for teachers in secondary schools. Professor W. D. Reeve: Teaching and supervision of mathematics: junior high school; Teaching and supervision of mathematics: senior high school. Professor Carl N. Shuster: Modern business arithmetic, methods of teaching in junior and senior high schools; Field work in mathematics. Dr. R. R. Smith: Mathematics for the eleventh year of the high school; Professionalized subject matter in geometry. Miss Ethel Sutherland: Teaching arithmetic in primary grades, first three grades, July 7 to 25; Teaching arithmetic in intermediate grades, fourth, fifth, and sixth grades, July 28 to August 15; Professionalized subject matter in junior high school mathematics.

Dr. A. Helen Tappan, Professor of Mathematics at Western College, Oxford, Ohio and Dean of Women since 1927 has been appointed Academic Dean succeeding Dr. Alice Hill Byrne who retires in June. Dr. Tappan is an alumna of Western College.

The Fourth Edition of R. C. Archibald's "Outline of the History of Mathematics" listed under New Books Received in the January (1940) issue of THE MATHEMATICS TEACHER is exhausted, but a new edition will be ready by September 1941.

On St. Valentine's Day, February 14, at 6:15 p.m. the Women's Mathematics Club were hostesses to the Men's Mathematics Club at the yearly joint meeting of the two clubs.

Miss Evelyn H. Roberts spoke on Mathematics as applied to her work in the technical laboratories of Sears Roebuck and Company.

At the speakers table Miss Clara Haertel of Englewood High School and her fellow officers, welcomed guests of honor, Miss Mary Potter, of Racine, Wisconsin, President of the National Council of the Teachers of Mathematics; Mr. Haris Gutekunst of Batavia, Illinois; Mr. Charles E. Jenkins of Foreman High School and S. S. Bibb of the Illinois Institute of Technology; President, Secretary-treasurer and Recording-secretary, respectively, of the Men's Mathematics Club.

Twenty-five hostesses from as many schools in Chicago and Vicinity presided at tables decorated with mathematical figures and puzzles, made by mathematics pupils of these teachers.

LENORE KING
Publicity chairman

The San Joaquin Valley Section of the California Mathematics Association voted to become affiliated with the National Council of Teachers of Mathematics at a recent meeting. Dr. Frank R. Morris of the Fresno State College is president of the Section, and Miss Ethel Spearman is secretary. The section had the following program:

Preliminary Report of the State Committee on Secondary Mathematics, Donald W. Larwood, Roosevelt High School, Oakland.

The National Council of Teachers of Mathematics, Miss Emma Hesse, University High School, Oakland.

Mathematics and the Defense Program, Professor H. M. Bacon, Stanford University.

Luncheon was served at the Commercial Club. The following list gives the members of the section:

Fresno County

Wynette Fowler, Caruthers
Paul E. Andrew, Clovis Union High School
Lillian Almquist, 821 Palm Avenue, Fresno
Ione Fox, Rt. 8, Box 130, Fresno
Library Fresno High School
Library Fresno State College
Donald W. Larwood, 324 Princeton, Fresno
Dr. Frank R. Morris, Fresno State College
Harry Renaud, 1449 F Street, Reedley
Clarence Irwin, Box 134, Riverdale
Ethel Spearman, Box 315, Sanger
Tranquility Union High School
Alice K. Bell, Fresno State College
Mrs. Adelia F. Bowlen, 3611 Platte Avenue Fresno
Miss Mabel Grandsband, 225N Fulton, Fresno

Kern County

Eileen Bowling, Bakersfield
Kern County Union High School
Margaret Martinson, Bakersfield
Edith Lee McLean, Bakersfield
Helen M. Plaum, Bakersfield
Dr. J. M. Robb, Taft Junior College

Madera Couhty

Grace Fuller, Madera

Tulare County

Dinuba High School, Dinuba

Mariposa County

C. G. Adelsbach, Mariposa

The Mathematics Association of Southwestern Illinois, The Southwestern Division of the Illinois Education Association, consists of twelve counties, namely, St. Clair, Madison, Bond, Calhoun, Green, Jersey, Clinton, Marion, Jefferson, Washington, Monroe and Randolph. They are having their meetings this year. The group is much interested in becoming affiliated with the National Council of Teachers of Mathematics.

The present officers are Miss Christine Fischer, Township High School, Belleville, President; H. E. Fletcher, Community High School, Granite City, Vice president, and Miss Arpha Holmes, Community High School, New Athens, Secretary-Treasurer.

The Autumn Meeting of the Mathematics Section of the Minnesota Education Association was held in St. Paul on Thursday, October 24, 1940.

General Theme: "Mathematics for the New Era"

President: Eleanor C. Biebl, Marshall, Minnesota

12:00 m. Luncheon
Speaker: Ella C. Clark, Winona State Teachers College, Winona, Minn. "Visual Aids in Mathematics"

2:00 p.m. Junior High School and Elementary Arithmetic Section
Clarence Eder, Owatonna, Minn. "Securing Interest in Ninth Grade Algebra"
Earl Albert, Winona, Minn. "The Nature and Purpose of Consumers Mathematics"
Florence Nibbe, Northfield, Minn. "The Use of Life Situations in the Classroom"
Ella M. Probst, Minneapolis, Minn. "Developing an Appreciation of the Power, Beauty, and Worth of Mathematics."

2:00 p.m. Senior High School Section
George McCutcheon, New Ulm, Minn. "Mathematics for the Non-College Pupil"
Edna Norskog, Houston, Minn. "Devices for Stimulating and Maintaining Interest in the Classroom"
Harvey Jackson, Minneapolis, Minn. "Making Use of Community Resources in Teaching Mathematics"
Ethel Saupe, Tracy, Minn. "Individual Differences—How Determined and How Solved"

3:00 p.m. General Meeting
Ralph K. Watkins, Professor of Education, University of Missouri "Generalized versus Specialized Mathematics in the Modern Secondary School"

LUELLA E. LEETE, *Secretary*

The North San Joaquin Mathematics Teachers Association met in Stockton, California, on Jan. 18, 1941.

Col. Wm. Pyle of the Air Corps School in Stockton spoke on "The Mathematical Preparation of Air Corps Cadets."

Present as guests were Miss Emma Hesse, State Representative of the National Council;

Dr. Sophia Levy of the University of California; and Mrs. Ruth G. Sumner of Oakland High School in Oakland.

These officers for 1941 were elected:

President: Miss Jennie Cowan, Modesto High School

Vice-president: Miss Louise Kemp, Oakdale High School

Secretary-treasurer: Miss Edith Chidester, Stockton High School

The organization took steps to affiliate with the National Council.

A. Gordon, *Secretary-treasurer*, 1940

ATTENDANCE AT BATON ROUGE MEETING

Dec. 30, 1940-Jan. 1, 1941

Compiled by Edwin W. Schreiber, Secretary

Alabama
- Birmingham
  - Moore, Wesley A.
- Boaz
  - Garrison, Lester M.*
- Decatur
  - Speer, E. E.*

Arizona
- Flagstaff
  - Mehlenbacher, Lysle E.

Arkansas
- Arkadelphia
  - Dorroh, J. L.
- Fayetteville
  - Richardson, Davis P.*
- Helena
  - Hudgens, Addie Beth*
- Mena
  - Middleton, W. E.

Colorado
- Boulder
  - Kemper, Audrey J.*
- Denver
  - Charlesworth, H. W.*
  - Charlesworth, Mrs. H. W.
  - Marinoff, Oscar

District of Columbia
- Washington
  - Grubbs, Ethel Harris*
  - Schult, Veryl*

Florida
- Miami
  - Stith, Estelle
- Tallahassee
  - Tew, Thelma

Georgia
- Carrollton
  - Scarborough, Henry B.

Illinois
- Chicago
  - Bibb, Samuel F.*
  - Fogelson, Ida D.*
  - Lytle, Edith*
- Dundee
  - Innes, Frances C.*
- Evanston
  - Moulton, E. J.
  - Wescott, Mason E.
- Freeport
  - Baumgartner, Reuben*
- Macomb
  - Ayre, H. G.*
- Mason City
  - Moure, Marian
- Monticello
  - Seybold, Anice*
- Springfield
  - Raman, Blanche*
- Urbana
  - Blackwell, David
  - Hartley, Miles C.*
- Waukegan
  - Barczewski, Walter*

Indiana
- Culver
  - Obenauf, H. A.
- Muncie
  - Edwards, P. D.*
  - Whitcraft, L. H.*

Iowa
- Ames
  - Gowens, Cornelius
- Cedar Falls
  - Kearney, Dora E.*
- Davenport
  - Hoskins, Catharine
- Des Moines
  - Carl, Florence*
- Keokuk
  - Nickle, George*
- Osage
  - Larson, Ira*
- Waterloo
  - McKinley, Eula*

Kansas
- Emporia
  - Peterson, Oscar J.*
- Manhattan
  - Babcock, Rodney W.
  - Daugherty, R. D.*
  - Stratton, W. T.*
- Pittsburg
  - Shirk, J. A. G.*
- Wichita
  - Beito, Edwin A.*
  - Read, C. B.*

Kentucky
- Bowling Green
  - Gilbert, Dawn*
  - Tryphena, Howard*
- Cadiz
  - Blakeley, Laura*
- Lexington
  - Wright, Harvey A.
- Louisville
  - Wood, Edith*
- Valley Station
  - Owings, Mrs. Mildred*

Louisiana
- Amita
  - Ramsey, Ruth E.
- Baton Rouge
  - Highland School
    - Kennedy, Lillian C.
    - Polizzotto, Myrtie
  - Istrouma School
    - Garrett, Mrs. Hattie
    - Rosenthal, Carolyn*
  - L. S. U.
    - Blanchard, Dorothy
    - Bordelon, W. J.
    - Christensen, Annie*
    - Cole, J. P.
    - Karnes, Houston T.*
    - Lawrence, W. A.*
    - Ramsey, A. K.
    - Rickey, Frank A.
    - Salter, J. J.
    - Saunders, S. T.*
    - Scott, P. C.*
    - Yates, Robert*
  - N Highland Jr. High
    - Brian, Catherine*
  - University Lab. School
    - Deer, George H.
    - Noah, D. P.*
- Bella Rosa
  - Turner, Donald*
- Chataignier
  - Rozas, Joseph*
- Delhi
  - Warden, Mrs. Meta*
- Ferriday
  - Davis, Mrs. Edith
- Hammond
  - Cordrey, William A.*
  - Tucker, B. A.*
- Iowa
  - Jackson, Alfred
- Jennings
  - Hoag, Jessie May*
- Kaplan
  - Montomgery, Thyra
- Lafayette
  - LeBlanc, Melva*
- Monroe
  - Currie, John Cecil
- Natchitoches
  - Gallion, L. T.
  - Maddox, A. C.*
- New Orleans
  - Crosby, Maurice H.
  - Discon, Mercedes C.
  - Forno, Dora M.*

Koch, Anna F.
Lively, Emery C.
Mayer, Joanna
Menuet, Robert L.
Sherrard, Erin*
Wischan, Caroline
Oakdale
Scott, Ora M.*
Oberlin
Rodgers, Alma May
Port Hudson
Devall, Berthalla
Ruston
Jones, Grady E.
Kaltenborn, H. S.
White, Daisy*
Scotlandville
Brantley, Edward
Hudson, S. M. (Mrs.)
James, William H.
McLeod, J. W.
McLeod, Myrtle D.
Marshall, George
Posey, L. R.
Smith, Russell W.
Williams, Willie
Shreveport
Friedenberg, Edgar

MARYLAND
Balto
Lane, Florence R.
McDonnal, Georgia W.

MASSACHUSETTS
Boston
Sherman, Ruth M. (Mrs.)*

MICHIGAN
Niles
Champion, Ella
Swan, Edna

MINNESOTA
Minneapolis
Hart, William L.*
Jackson, Dunham*
Woolsey, Edith*

MISSISSIPPI
Hattiesburg
Dearman, Dewey S.*
Jackson
Barton, James
Beckett, James*
Lester, Annie*
McCoy, Dorothy
Marshall, Vivian
Spann, Pearl*
Vance, Virginia*
Oxford
Lindsey, Clyde*
Raymond
McDonald, Janet

Washington
Dodson, Norman*
University
Bickerstaff, T. A.

MISSOURI
Columbia
Doolittle, Nellie*
St. Louis
S. M. Patricia Calleghan
Sister Jeanne D'Arc Hurley
Springfield
Beasley, Louise*

NEBRASKA
Kearney
Hanthorn, Emma E.*
Lincoln
Clark, Myrtle E.*
Congdon, Allan R.*
Cook, Inez M.*

NEW JERSEY
New Brunswick
Morris, Richard*

NEW MEXICO
State College
Delebarty, R. D.

NORTH CAROLINA
Boone
Wright, J. T. C.*
Pinehurst
Tilley, Edward B.*

NEW YORK
Alfred
Seidlin, Joseph*

OHIO
Cincinnati
Barnett, Mrs. Fannie
Oberlin
Cairns, W. D.*
Oxford
Christofferson, H. C.*

OKLAHOMA
Norman
Hassler, J. D.*
Tulsa
Gartman, Stella*
Wright, Etoile*

PENNSYLVANIA
Allentown
Deck, Luther J.*
Chambersburg
Asbury, Jean*
Pittsburgh
Olds, Edwin G.*

SOUTH CAROLINA
Columbia
Coleman, Jas. B.*
Laurens
Johnson, H. C.
Thompson, F. P.*
Rock Hill
Stokes, Ruth W.*
Sumter
Herbert, Harriet*

TENNESSEE
Alcoa
Howard, Allene*
Monterey
Nixon, Milton*
Nashville
Miser, Wilson*
Wright, M. Iammack
Wren, F. Lynwood*

TEXAS
Arlington
Howard, C. M.
Brownwood
Freese, Frances*
Dallas
Dice, Elizabeth*
Holder, Mrs. Lorena*
Denton
Hanson, Eugene H.*
El Paso
Harris, Leila Mae
Port Arthur
King, Sue
Kirkham, John N.
Turner, Nura*
San Antonio
Sister Anacletus Moore

WASHINGTON
Bellingham
Bond, E. A.*
Spokane
Bell, Kate*

WEST VIRGINIA
Montgomery
Maple, Clair
Smith, Wallace*
Smith, Mrs. Wallace
Smithers
Milo, Gasperine*

WISCONSIN
Green Bay
Rueppel, Eunice A.*
Swan, Jessie*
Milwaukee
Felice, Sister Mary*
Racine
Pottery, Mary A.*
Superior
Person, Ruth*

* Denotes membership in the National Council of Teachers of Mathematics.

NOTICE

The Private Schools Association of the Central States Mathematics Section will hold a meeting on May 3, 1941 at 10 A.M. at the Lake Forest Day School at Lake Forest, Illinois.

ATTENDANCE BY STATES

STATES	MEMBERS	NON-M.	TOTAL
Ala.	2	1	3
Ariz.	0	1	1
Ark.	2	2	4
Colo.	2	2	4
D. C.	2	0	2
Fla.	0	2	2
Ga.	0	1	1
Ill.	10	4	14
Ind.	2	1	3
Iowa	5	2	7
Kans.	6	1	7
Ky.	5	1	6
La.	21	36	57
Md.	0	2	2
Mass.	1	0	1
Mich.	0	2	2
Minn.	3	0	3
Miss.	9	5	14
Mo.	2	2	4
Nebr.	4	0	4
N. J.	1	0	1
N. M.	0	1	1
N. Car.	2	0	2
N. Y.	1	0	1
Ohio	2	1	3
Okla.	3	0	3
Pa.	3	0	3
S. Car.	4	1	5
Tenn.	4	1	5
Texas	5	5	10
Wash.	2	0	2
W. Va.	2	2	4
Wis.	5	0	5
Total	110	76	186